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ABSTRACT

This instructional unit focuses on writing ratios and proportions in problem situations, solutions by means of proportions, and determination of percentages. A number of experiments are suggested and worksheets and discussion questions are included. The activities are oriented toward situations in which the students would probably have had some previous experience. A teacher's guide is also available. Related documents are SE 015 334, SE 015 335, and SE 015 337 through SE 015 347. This work was prepared under an ESEA Title III contract. (LS)

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ACTIVITIES WITH RATIO & PROPORTION

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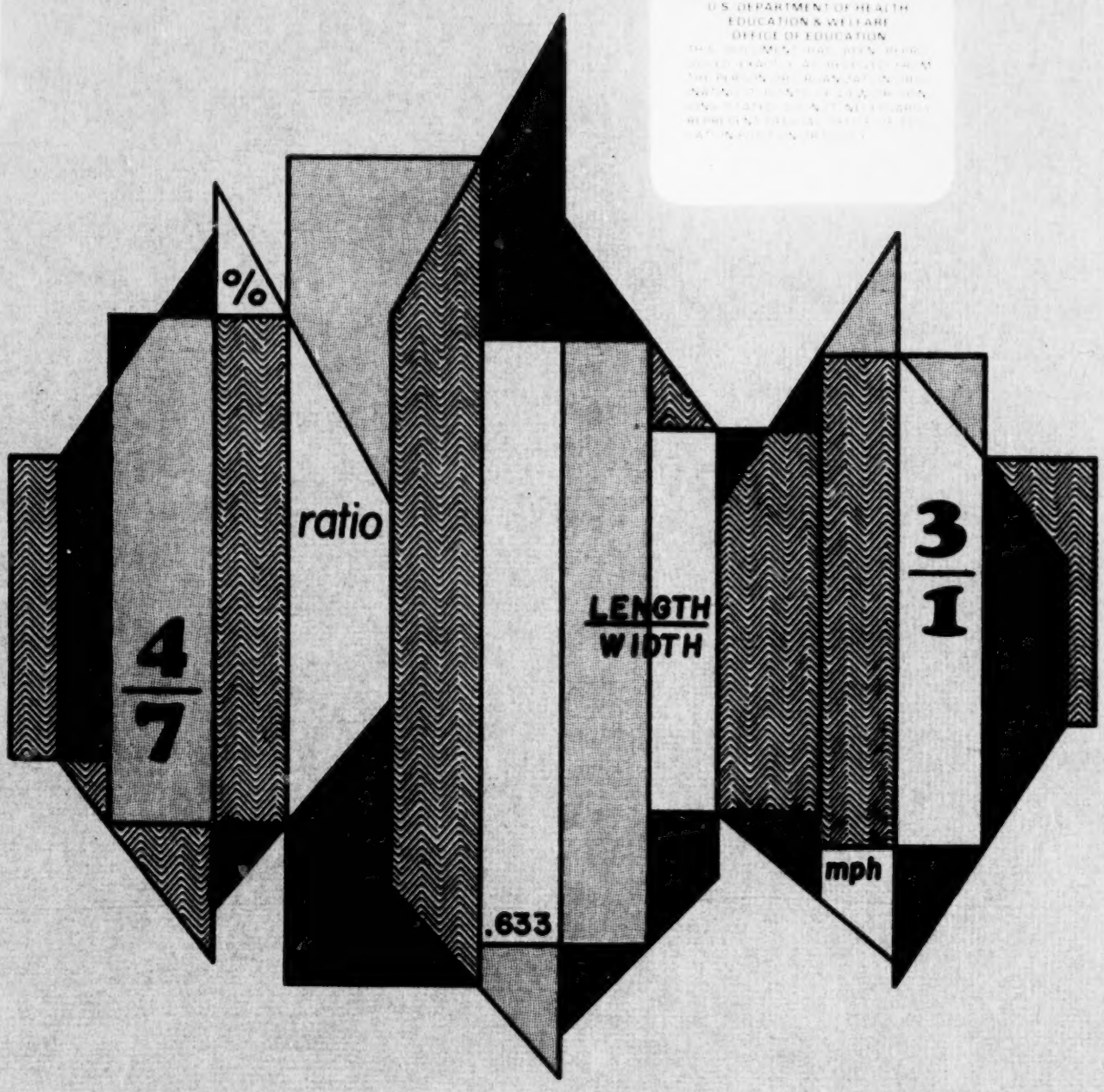

SE 015 336

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ED 069525

ACTIVITIES WITH RATIO AND PROPORTION

OAKLAND COUNTY MATHEMATICS PROJECT

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PREFACE

Ratios and proportions are useful tools for problem solving. They can be used in many situations involving numerical relations. Some examples are comparing winning records of baseball teams, predicting the weight of an object suspended on a spring, and increasing the size of a recipe to feed a large number of people. It is because of the problem-solving power of ratio and proportion that this booklet was written.

If a technician working for a manufacturing firm knows the weight and volume of a metal object, he can tell what it is made of. This identification process (known as finding specific gravity) can be used by a detective (What is this piece of metal?), a dairy worker (Does the milk meet the state butterfat content requirement?), a service station attendant (Is there enough antifreeze in the radiator? Is this battery fully charged?), as well as in many other areas.

Each lesson centers on one or two main ideas. This booklet contains a variety of activities to help you develop and apply the ideas of ratio and proportion. Read, listen, and above all, participate in the discussions and class activities.

After actively participating in the activities of this booklet, you should be able to:

1. Write ratios and draw diagrams which represent statements like "7 out of 9 tires are whitewalls" and "For every 3 squares there are 5 triangles."

(LESSON 1)

2. Write ratios indicated by written problems.

(LESSON 2)

3. Determine whether or not two ratios are equivalent using paper folding or drawing diagrams.

(LESSON 3)

4. Write a ratio when a problem situation is read aloud to you.

(LESSON 4)

5. Simplify a given ratio.

(LESSON 5)

6. Determine if a proportion is true or false.

7. Solve a proportion for the missing member.

8. Write and solve a proportion suggested by a given problem situation.

(LESSON 7)

9. Use data resulting from an experimental activity to predict the outcome of a future trial of the experiment.

(LESSONS 6, 8, 10, 11, 17)

10. Use the cross-product method to determine whether or not two ratios are equivalent.

(LESSON 9)

11. Use proportions and per cent to solve problems like the following.

    a. Use a proportion and 100 as a basis to compare costs and athletic performances.

    b. Determine the amount of medicine required to mix 40 oz. of a 35% solution.

(LESSON 12)

12. Estimate a whole number per cent of a given line segment.

13. Write a per cent in its various forms, such as 40% , $\frac{40}{100}$, .40 , and $\frac{2}{5}$.

(LESSON 13)

14. Estimate a whole number per cent of a given number.

(LESSON 14)

15. Use proportions to calculate a whole number per cent of a given number.

(LESSON 15)

16. Recognize whether or not a problem situation requires an inverse proportion.

17. Solve problems requiring inverse proportions.

(LESSON 16)

You should be able to apply what you have learned about ratio and proportion to other booklets this year as well as in your future mathematics and science classes.

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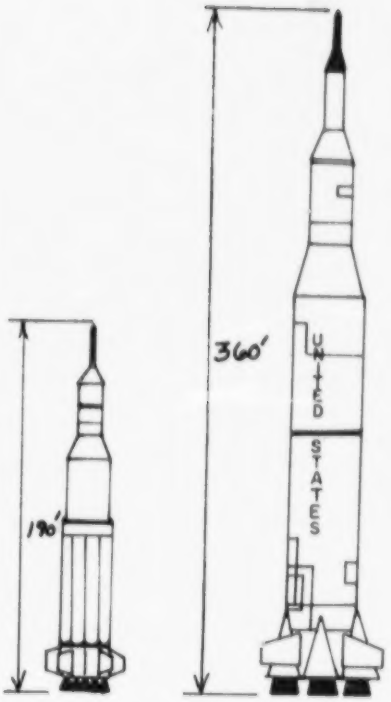
THREE OUT OF EVERY FOUR. . .

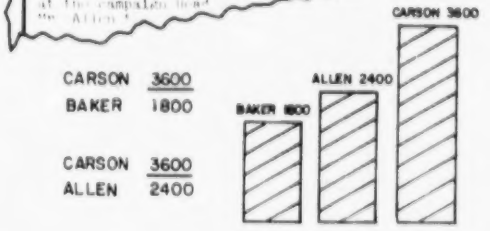


BOX SCORE

TOTAL GAMES	30
WINS	19
LOSSES	11
	.633
PITCHING RECORD	

CARSON WINS ELECTON OVER ALLEN

MIDWEST CITY - Mr. A. W. Carson was elected Mayor of Midwest City today. He defeated his two opponents with a vote of 5400. Mr. P. G. Baker received a vote of 1800 while ex-Mayor R. C. Allen failed in his reselection bid with a vote of only 2700. Although Mr. Carson had the majority of the vote he failed to obtain at least 50 per cent of the popular vote. He appeared at the campaign head- quarters of Allen's



CARSON	3600
BAKER	1800
CARSON	3600
ALLEN	2400

A baseball pitcher's record can be written as a ratio. The ratio is $\frac{19}{30}$ where the numerator is the number of wins and the denominator is the total of wins and losses. The decimal .633 is found by dividing 19 by 30. This is the way a pitcher's record is reported in the newspaper.

Stores offer sales to clear out goods they have had on hand in order to make room for new merchandise. The amount items are reduced is usually figured by using per cent. What does it mean when a store advertises 15% off? How are per cents written as ratios?

The results of an election are reported using the number of votes each candidate receives. The person receiving the highest total wins the election. Who was the winning candidate? By how many votes did he beat his nearest rival? Ratios can be used to compare votes. The ratio $\frac{3600}{1800}$ shows that people voted 2 to 1 in favor of Mr. Carson over Mr. Baker. What ratio indicates the comparison of votes for Mr. Carson to votes for Mr. Allen? What per cent of the vote did Mr. Carson get?

Saturn I was one test rocket used to explore space before an actual moon flight was attempted using the Saturn V. What is the ratio of the height of Saturn V to the height of Saturn I? About how many times longer is Saturn V? What is a ratio that compares the number of engines? Does it seem reasonable that a longer space rocket uses fewer engines?

Ratios are used in many situations, some of which are indicated on the previous two pages. Performances of cars, athletes, students, and home appliances can be compared using ratios.

PART I

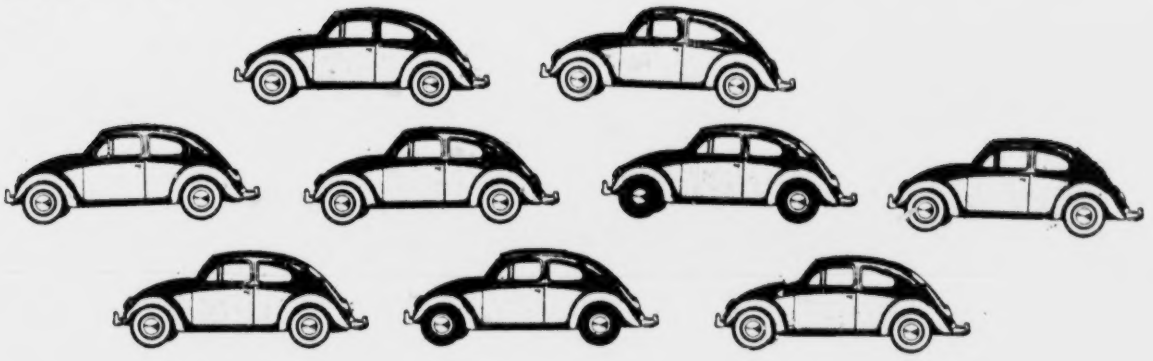
The first activity of this unit contains a review of the meaning of ratio. Your teacher will use the overhead projector to illustrate several ways ratios can be interpreted.

Watch carefully and participate in the discussion. If you have questions during the discussion, ask them.

PART 2

There are several kinds of statements indicating that a ratio could be written. In this part, you will work with one kind of statement.

EXAMPLE 1:



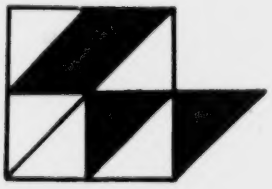
7 out of the 9 cars have white-walled tires.

$\frac{7}{9}$ expresses the statement in ratio form.

The denominator of the ratio represents the entire set.

The numerator represents the part of the set referred to.

EXAMPLE 2: 4 out of 9 triangular regions are shaded.



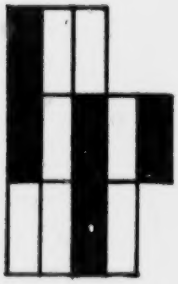
4 out of 9 is represented by $\frac{4}{9}$.

The denominator of the ratio is 9, the numerator is 4.

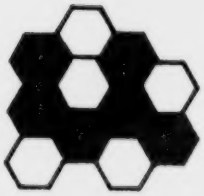
EXERCISES

1. 2 out of 5 triangular regions are shaded. Write the ratio represented by the statement.

2. 4 out of 7 square regions are shaded. Write the ratio.

3. 

Of the 12 rectangular regions, 5 are shaded. Write the ratio indicated.

4. 

a. 6 out of the 11 hexagonal regions are shaded. Write the indicated ratio.

b. 5 out of the 11 hexagonal regions are <u>not</u> shaded. What ratio represents this statement?

5.

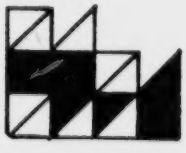
a. ____ out of 6 circular regions are shaded.

b. Write the ratio.

6.

a. 4 out of ____ squares are circled.

b. Write the ratio.

7. 

a. ____ out of _____ triangular regions are shaded.

b. Write the indicated ratio.

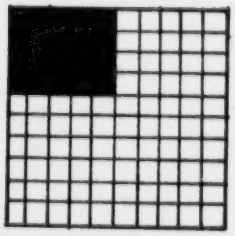
8.

a. Circle the correct number of triangles to illustrate: 7 out of 9 triangles are circled.

b. Write the indicated ratio.

✓ **POINT**

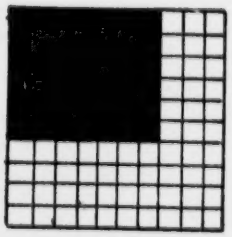
How do you know which number is the numerator and which is the denominator?

9. 

The picture contains 100 squares like this: □

a. ____ out of the 100 square regions are shaded.

b. What ratio is indicated?

10. 

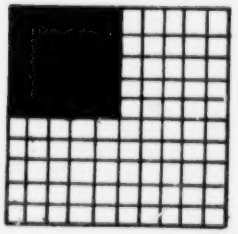
a. ____ out of 100 regions are shaded.

b. Write the ratio indicated.

c. ____ out of ____ regions are <u>not</u> shaded.

d. What ratio is represented now?

25 out of 100 regions are shaded. $\frac{25}{100}$ is the indicated ratio. Ratios having a denominator of 100 are given the special name PER CENT. Per cent means "out of 100" or "for each 100".



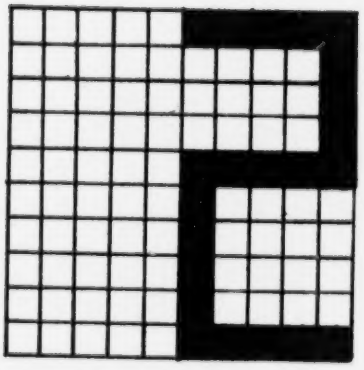
25 per cent and $\frac{25}{100}$ represent the same idea.

11. a. Write 35 per cent as a ratio.

    b. $\frac{75}{100}$ can be written as ____ per cent.

    c. If 40 out of 100 regions are shaded, the ratio indicated is ___ . Write the ratio using its special name.

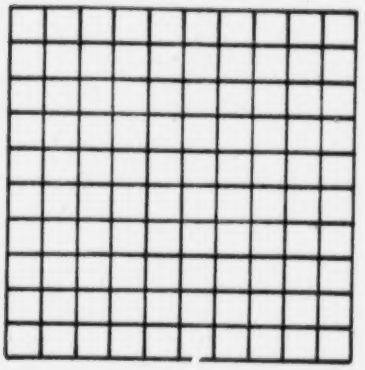
12.



a. In the picture, ____ out of 100 regions are shaded.

b. The ratio indicated is ____.

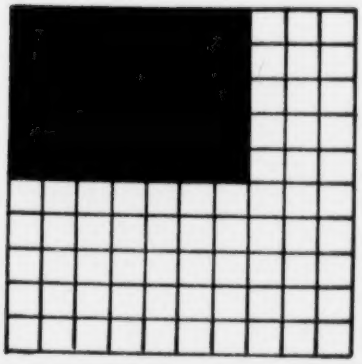
c. Express the ratio as a per cent.

13. 

a. Shade the small square regions to show that 15 out of 100 regions are shaded.

b. What ratio is represented by the picture?

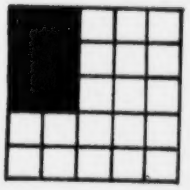
c. Write the ratio as a per cent.

14. 

35 per cent of the regions are shaded.

Circle below the ratio representing the statement:

$\frac{35}{65}$ $\frac{65}{100}$ $\frac{100}{35}$ $\frac{35}{100}$

15. 

a. ___ out of ____ regions are shaded.

b. The indicated ratio is ____.

c. At the same rate, how many regions would be shaded out of a total of 50 regions?

16.

a. ____ out of ____ regions are not shaded.

b. Write the indicated ratio.

17. 4 out of 10 triangular regions are to be shaded.

a. Complete the picture:

b. Write the ratio.

18. a. Complete the picture to represent $\frac{6}{10}$:

b. ____ out of ____ circular regions are shaded.

19. In a picture using circles to represent $\frac{4}{11}$,

a. How many circles would be in the entire picture?

b. How many circular regions would you shade?

c. Draw a picture to represent the ratio $\frac{4}{11}$.

20. Which of the ratios represents the statement, 18 out of 53 triangular regions are shaded?

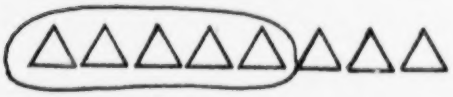
$\frac{53}{18}$ $\frac{18}{100}$ $\frac{18}{53}$ $\frac{53}{100}$

PART 3

The ratio $\frac{5}{8}$ can be used to express the statement "For every 5 there are 8." This statement differs from the first type we wrote because it indicates that two different sets of objects are being compared. The first type of ratio was a comparison of part of a set to the whole set.

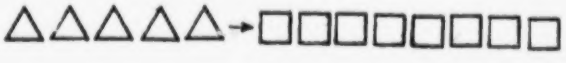
Look at the two types of ratios to note their differences:

5 out of 8 triangles are circled.



$\frac{5}{8}$

For every 5 triangles there are 8 squares.



$\frac{5}{8}$

EXAMPLE: Draw the picture and write the ratio representing: For every 11 * there are 6 #.

Answer:

Picture	Ratio
* * * * * * * * * * * → # # # # # #	$\frac{11}{6}$

✓ **POINT**

How do the ratio statements and pictures introduced in Part 3 differ from those you worked with in Part 2 ?

EXERCISES

1. For every 2 △ there are 5 □.
   a. Complete the picture: △△ →
   b. Write the ratio which represents the statement.

2. For every 3 △ there are 7 □.
   a. Complete the picture: → □□□□□□□
   b. Write the ratio representing the statement.

3. For every 4 △ there are 5 □.
   a. Draw a picture representing the statement.
   →
   b. Write the ratio.

4. Use the picture to fill in the blanks: For every _____ there are _____.

   △△△△ → □□□□□□□

5. Consider a set of 5 ○ and a set of 6 ⬡.
   a. Write a ratio indicating the number of ○ compared to the number of ⬡.
   b. Draw a picture which represents the ratio.
   →
   c. Fill in: For every _____ there are _____.

6. For every 3 ◯ there are 7 ⬡.

   Circle the ratio which represents the above statement. $\frac{3}{7}$ $\frac{7}{3}$

7. Given 4 ⬡ and 9 ◯ and the ratio $\frac{9}{4}$, fill in the blanks:

   For every _____ there are _____ .

8. Draw a picture representing the statement: For every 6 □ there are 11 △ .

   a. →

   b. What ratio represents the statement?

9. For every 7 △ there are 4 □ .

   a. Complete the picture: → □□□□

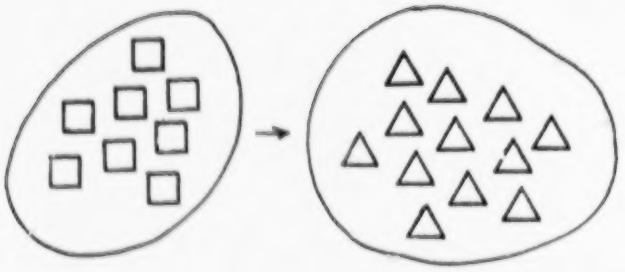
   b. Write the ratio.

10. Consider the ratio, $\frac{3}{5}$.

    a. Draw a picture representing the ratio.

    b. For every _____ there are _____.

11. Look at these two sets:



a. For every 8 □ there are _____.

b. Write a ratio comparing □ to △ .

12. For every 8 △ there are 10 ○ .

a. Complete the picture

△△△△△△△△ →

b. Write a ratio which represents the picture.

13. Write the ratio represented by

□□□□ → △△△△△△△

14. For every 6 ○ there are 9 □ .

a. Complete the picture: → □□□□□ □□□□

b. Write a ratio which represents the picture.

c. This exercise compares the number of _____ to the number of _____.

For every 2△ there are 3⬡ is expressed by the ratio $\frac{2}{3}$.

△△ → ⬡⬡⬡

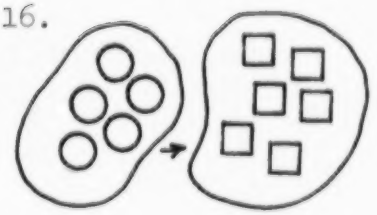
Another statement can be written for the picture. It is "2 triangles compared to 3 hexagons." The ratio $\frac{2}{3}$ also represents this.

Example: There are 28 days in February compared to 31 days in March. The ratio representing this comparison is $\frac{28}{31}$.

15. There are 4 triangles compared to 7 circles.

a. Complete the picture: △△△△ →

b. Write the indicated ratio.

16. 

a. 5 circles are compared to ____ squares.

b. The ratio is _____.

17. There are 7 * compared to 11 #

a. Draw a picture representing the comparison. →

b. Write the ratio.

c. Complete the statement: For every ____ there are ___ .

18. 5 out of 9 triangles are shaded.

    a. Write the ratio indicated by the statement.

    b. Draw a picture representing the ratio.

19. Write the ratio and draw a picture representing each of the following:

    a. For every 9 squares there are 6 triangles.

    b. 6 out of 9 squares are shaded.

    c. There are 9 triangles compared to 6 squares.

20. Circle the ratio which represents each of the following:

    a. There are 16 squares compared to 11 triangles.

    $\frac{11}{16}$ $\frac{11}{27}$ $\frac{16}{11}$ $\frac{27}{16}$

    b. For every 8 shaded triangles there are 15 circles.

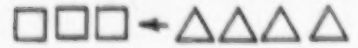
    $\frac{15}{8}$ $\frac{8}{15}$ $\frac{8}{23}$ $\frac{23}{15}$

    c. 9 out of the 17 squares are shaded.

    $\frac{17}{9}$ $\frac{9}{17}$ $\frac{9}{26}$ $\frac{8}{26}$

DISCUSSION QUESTIONS

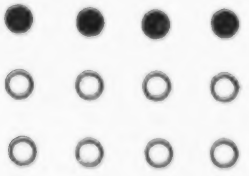
1. Look at the two pictures: A

   B 

   a. How do they differ?

   b. What ratios do they represent?

   c. Are both pictures correct? Explain.

2. Look at the drawing. 

   a. Write all of the different ratios and sentences that could describe the drawing.

   b. Was it possible to use a statement like

      For every ____ there are ____ , or ____ out of ____.

      to describe any of your ratios? Explain your answer.

3. a. If you were to close your eyes and imagine a picture for each of the statements, what would you see?

      2 out of 3 cars are red.
      For every 2 red cars there are 3 blue cars.

   b. Are the ratios different?

   c. Tell how the statements describe different situations.

CARDS, CARS, AND RATIOS

Ratios express relationships in a variety of situations. In this lesson you will be writing more ratios. These ratios come from many topics that you might find interesting.

EXERCISES I

You will receive cards containing some facts and questions about these facts. An objective of this exercise is for you to be able to write ratios which represent certain data given to you.

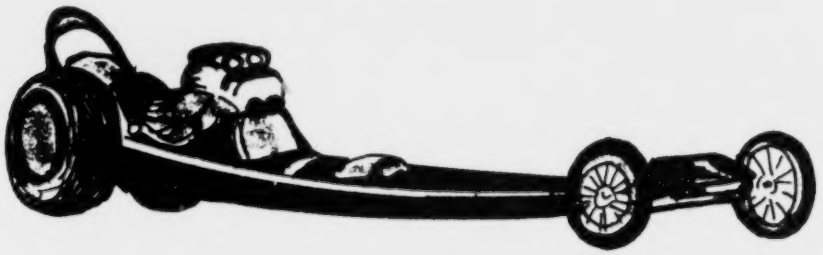
There are twelve cards in the set. Your teacher will explain how you are to do the exercises. Write your answers below.

CARD NUMBER	ANSWERS TO EXERCISES

EXERCISES 2

Do the following exercises using ratios:

1.



A dragster uses about 3 gallons of fuel for each run at a cost of $21.00.

a. Write a ratio that tells total cost per gallons of fuel for one run.

b. What does one gallon of fuel cost?

c. Write as a ratio the cost per gallon of fuel.

2.

A muscle in the human body is about as efficient as a good automobile engine.

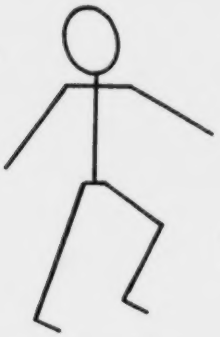
A muscle converts about 30 calories directly into energy out of every 100 calories used.

a. What ratio represents this comparison?

b. What % of the calories are used directly for energy?

3. During one minute of vigorous activity your lungs take in about 100 quarts of air. At rest your lungs take in about 8 quarts of air. Write a ratio comparing intake at rest to intake during activity.

4.



679 CALORIES

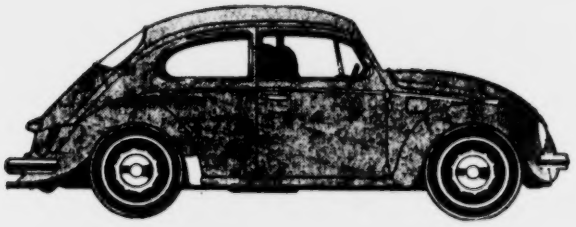
Walking uses 334 calories per hour. Jogging uses 679 cal/hr.

a. What is the ratio of calories used for jogging compared to calories used for walking?

b. The number of calories used for jogging is about how many times those used for walking?

334 CALORIES

5.



Today a gallon of gasoline can move a 2000 pound car at 40 mph nearly 40 miles.

35 years ago the same amount of gas would move the car about 30 miles.

Write a ratio comparing the miles traveled now to the miles traveled 35 years ago on a gallon of gas.

6. Most automobile trips are 10 miles or less.

It takes almost 7 miles to warm up an engine.

a. What fraction of a 10-mile trip is used for warm-up?

b. What is the ratio of warm-up distance to trip distance for a 10-mile trip?

The next exercises involve writing per cent as a ratio. Since % means "for each hundred" or "out of one hundred", 30% means 30 for each 100 (30 out of 100) and is written as a ratio like this: $\frac{30}{100}$. 95% is the ratio $\frac{95}{100}$.

7.

A maker of men's trousers stated that the material was 50% Dacron, 25% Orlon, 25% Rayon.

Write ratios that express these per cents:

Dacron: Orlon: Rayon:

8. A telephone company advertised that by dialing long distance direct you could save 40% on a three-minute call.

What ratio expresses this savings?

9.

A newspaper ad about tires stated that the supertread tire was 23% stronger than the average tire.

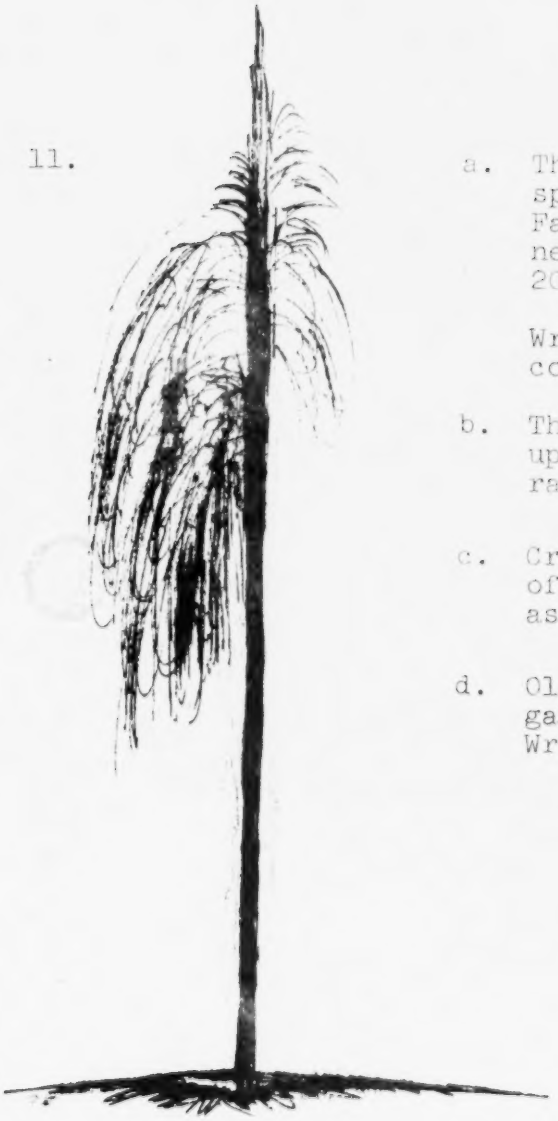
Express 23% as a ratio.

10. The newspaper ad also stated that this tire stops 25% more quickly than the average tire.

Write 25% as a ratio.

A car traveling 60 miles per hour is going 60 miles for each 1 hour of time. The ratio representing this is $\frac{60 \text{ miles}}{1 \text{ hour}}$. Another way to express this speed is 88 feet per second. Per second means "for each 1 second". The ratio would be written as $\frac{88 \text{ feet}}{1 \text{ second}}$.

11.



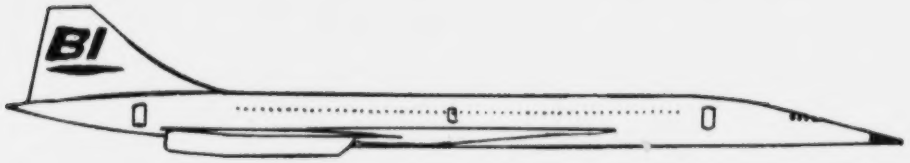
a. The Crump Geyser, in Oregon, is spraying water higher than Old Faithful's 160 foot spray. The new geyser sprays water and steam 200 feet in the air.

   Write a ratio of Crump's height compared to Old Faithful's height.

b. The Crump Geyser sprays at a speed up to 80 feet per second. What ratio represents this?

c. Crump also sprays up to 600 gallons of water per minute. Write this as a ratio.

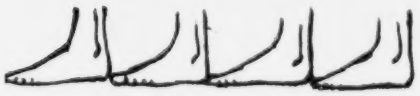
d. Old Faithful releases up to 2500 gallons of water per minute. Write the ratio.



12. Today a jet going from New York to London travels about 600 miles per hour. Write this as a ratio.

13. The new supersonic jets will travel at a speed of nearly 1800 mph. What ratio represents this?

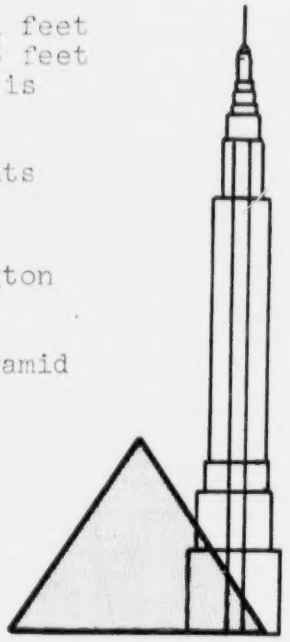
14. Today's jet going from New York to London uses a fuel at the rate of 13,000 gallons per trip. The new supersonic jets will use nearly 30,000 gallons per trip between these two cities. Write ratios to represent these ideas.

15.  Write as a ratio: There are 5280 ft. per mile.

16. Almost 8% of the earth's crust is aluminum. Write this as a ratio.

17. Write a ratio which represents that one acre of soil contains nearly 4 million insects.

18. The Egyptian pyramid (Cheops) is 481 feet tall; the Washington Monument is 555 feet high; and the Empire State Building is 1472 feet high.



Write ratios which compare the heights of the:

a. Pyramid of Cheops to the Washington Monument.

b. Empire State Building to the Pyramid of Cheops.

19. About how many times taller is the Empire State Building than the Egyptian Pyramid?

20. The oceans contain 97.3 per cent of the earth's water supply. Write this as a ratio.

21.

Two-cycle engines require a mixture of oil and gasoline. Some engines require 2 quarts of oil for every 5 gallons of gasoline.

Write a ratio representing the oil-gas mixture.

22. The table below shows how different tire aids improve stopping distances when a car is traveling 20 mph on glare ice.

Complete the table by writing each per cent as a ratio.

Tire aid for stopping	% Better stopping distance than with regular tires	Ratio
a. Studded Snow Tires (rear, new)	19%	
b. Studded Snow Tires (front, rear, new)	31%	
c. Tire chains	50%	

23.

A Sea Diver faces the problem of changing pressure on his body. To avoid danger, he must adjust the kind of air mixture he breathes and the speed he rises to the surface.

a. Air pressure at sea level is 14.7 pounds per square inch. Write this as a ratio.

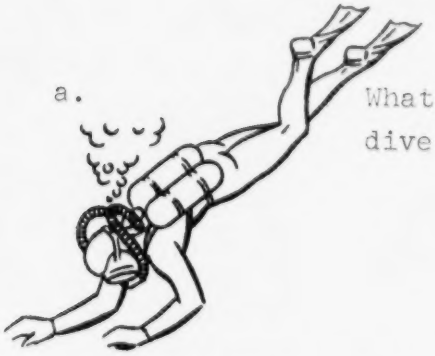
b. For every 33 feet he dives, the pressure is increased 14.7 pounds per square inch (psi). What ratio represents this?

c. At a depth of 33 feet, the pressure would be 29.4 psi. (14.7 psi at 0 feet + 14.7 psi for the depth of 33 ft. = 29.4 psi)

What would be the pressure at 66 ft. below sea level?

24.

Depth (ft.)	Event
0	Sea level
150	Normal skin diving limit
275	Deepest hard-hat salvage of a ship
300	Deepest skin dive breathing air
475	Deepest diving chamber salvage of a ship
4000+	Nuclear submarine depth
35,802	Deepest spot in the ocean



a. What is the ratio of the deepest skin dive to the normal skin diving limit?

b. What is the ratio of the deepest chamber salvage to the deepest hard-hat salvage?

A PICTURE OF EQUIVALENCE

EQUIVALENT RATIOS

There are many ratios which can represent the same picture. These ratios are said to be equivalent. Two questions that might be asked are: (1) How do you know when two ratios are equivalent? (2) How do you know if two ratios represent the same picture?

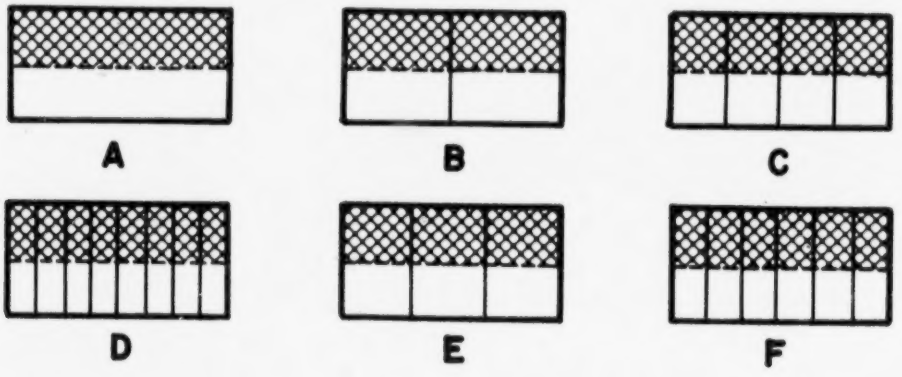
CLASS ACTIVITY

Your teacher will pass out some sheets of paper and discuss the activity. The purpose of the activity is to fold paper to illustrate equivalent ratios.

EXERCISES

The rectangles drawn below are like the rectangles you just worked with while paper-folding.

The rectangles are the same size overall, and the shaded regions are of equal size.



1. Figure A is divided into two rectangular regions. One of them is shaded.

   a. ____ out of ____ regions is shaded.

   b. Write the ratio.

2. Consider figure B.

   a. Into how many regions is figure B separated?

   b. How many of the smaller regions are shaded?

   c. Write the ratio that represents the situation.

3. Consider figure C.

   a. The number of small regions is ____ .

   b. ____ of the small square regions are shaded.

   c. What ratio represents the situation in figure C?

4. What ratio represents the situation in:

   a. figure D?

   b. figure E?

   c. figure F?

Each of the six pictures has the same area shaded, but different ratios have been used to describe the comparisons.

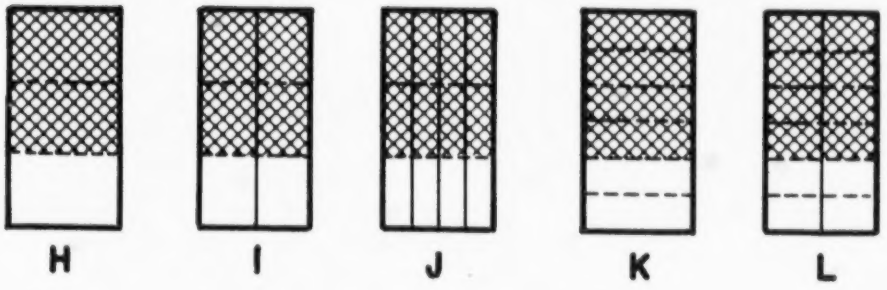
Different ratios describing the same situation (each rectangle had the same total area and the same amount of shading) are called EQUIVALENT RATIOS.

5. Write the six equivalent ratios described by pictures A, B, C, D, E, F.

6. a. Write the ratio representing the amount of shaded area to total area for each picture below:

b. Since each of the ratios you wrote in part a represent the same size regions, they are called _____ ratios.

7.



a. Picture H is represented by the ratio _____.

b. How do the sizes of the large rectangle compare?

c. How do the sizes of the shaded portions compare?

d. What term applies to the set of ratios representing pictures H - L?

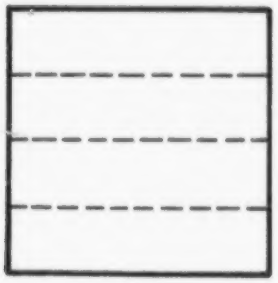
e. Write the ratios representing I: J: K: L:

✓ POINT

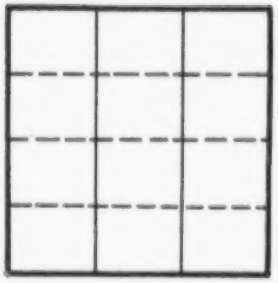
Suppose a picture having a shaded region is subdivided several times. As you further subdivide, what happens to the total area of the figure and to the area of the shaded region?

EXERCISES , CONT'D.

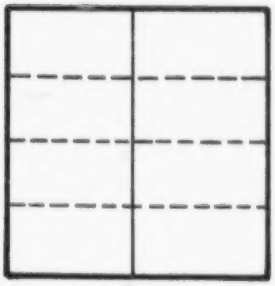
8.



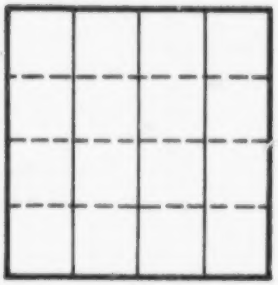
A



B



C



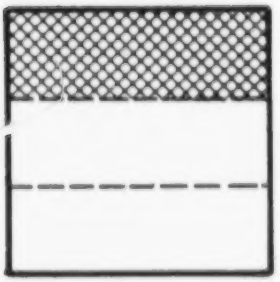
D

a. In picture A, represent the ratio $\frac{1}{4}$ by shading.

b. Shade the remaining diagrams according to the table to show that the ratios are equivalent.

Diagram	A	B	C	D
Ratio	$\frac{1}{4}$	$\frac{3}{12}$	$\frac{2}{8}$	$\frac{4}{16}$

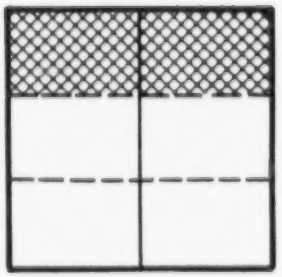
Example:



M

Figure M has been separated into 3 regions and one of them is shaded.

The ratio represented here is $\frac{1}{3}$.



N

Figure N has an additional line segment.

The total number of regions has been doubled, 6 in all.

Also the number of shaded regions has been doubled, 2.

The ratio represented here is $\frac{2}{6}$. $\frac{2}{6}$ is equivalent to $\frac{1}{3}$ because the overall figures and the shaded portions are equal in area.

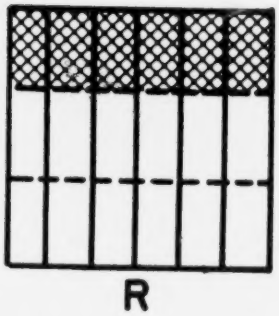


Figure R has 6 times the total number of regions as does figure M, 18.

Figure R also has 6 times as many shaded regions as figure M, 6.

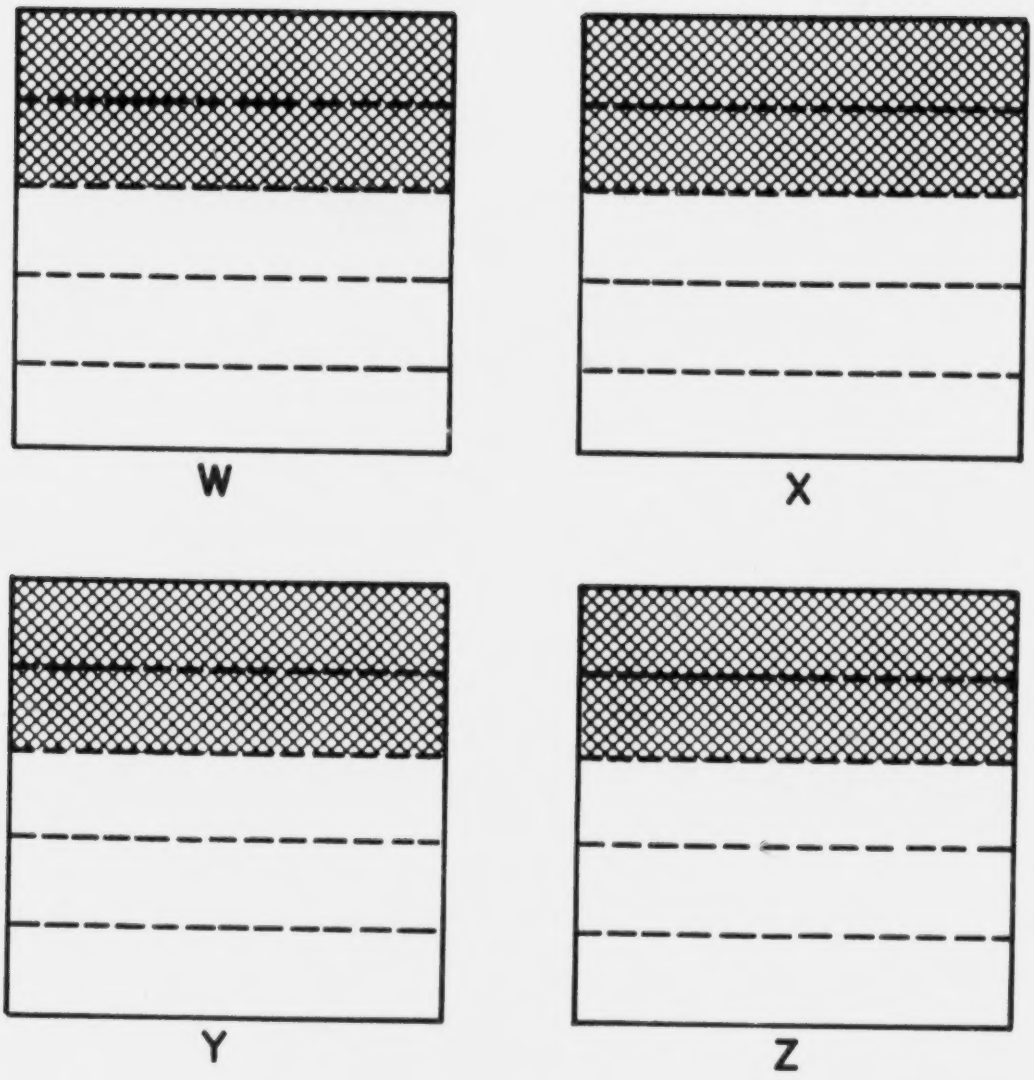
The ratio represented by R is $\frac{6}{18}$.

The example shows that $\frac{1}{3}$ is equivalent to $\frac{2}{6}$ and to $\frac{6}{18}$. That is, $\frac{1}{3} = \frac{2}{6}$ and $\frac{1}{3} = \frac{6}{18}$.

N and R have the same areas (total and shaded) so $\frac{2}{6} = \frac{6}{18}$.

The total number of regions in R is 3 times the number in N. Also the number of shaded regions in R is 3 times the number in N.

9.



In the pictures above, 2 out of 5 regions have been shaded, representing the ratio $\frac{2}{5}$.

Pictures X, Y, Z are to be changed to represent ratios equivalent to $\frac{2}{5}$ (picture W.)

a. Draw a segment in picture X so that the regions will be equal in area and represent $\frac{4}{10}$, a ratio equivalent to $\frac{2}{5}$.

b. What is the total number of regions in X now?

c. How many shaded regions are there in X?

d. Write the ratio represented by X.

e. Draw four vertical line segments equally spaced in picture Y. The total number of regions should be 25. How many shaded regions are there?

f. What ratio does Y represent now?

g. Draw vertical lines in picture Z to represent the ratio $\frac{6}{15}$.

h. Write the four equivalent ratios discussed in the exercise.

✓ POINT

1. Write the ratios indicated by the following:

   a. Four out of five girls wear rings.

   b. A sign read: "Shoe Sale, 15 per cent off".

   c. For every 6 boys there are 8 girls.

   d. & & & ⟶ # # # # # # #

   e. My test score was 85 per cent.

   f. An iceberg was estimated to be 200 ft. high above the water and 400 ft. below the surface. What ratio compares the distance under water to the distance above water?

   g.

2. How do you know if two ratios are equivalent or not?

3. Show that $\frac{2}{3}$ and $\frac{8}{12}$ are equivalent.

4. Complete the following to represent the ratio $\frac{4}{9}$.

   a. △△△△→

   b. ____ out of ____ are green.

   c. Shade: △△△△△△△△△

   d. For every____ there are ____ .

CANDLES AND RATIOS

PART I

Ratio and proportion is used a great deal when predictions need to be made. Many experiments involve proportional relationships. In later lessons you will be doing activities that use proportions.

Your teacher will demonstrate the activity, CANDLES AND JARS, as part of this lesson. The important things for you to think about are:

1. An equipment inventory should be made.
2. The entire activity should be read over so that the purpose and procedure are understood.
3. Do the activity carefully.
4. Follow the directions and make predictions.
5. Think about the results:
   a. Are they reasonable?
   b. What accounts for differences in the experimental results and the predicted results?
   c. Are these differences acceptable?
   d. Would there be any advantage in repeating the experiment several times?

PART 2

EXERCISES

This is an exercise in listening and writing. Your teacher will read aloud some information and then ask you to write ratios about it.

MORE ABOUT EQUIVALENCE

SIMPLIFYING RATIOS

Simplified ratios are used mainly because they help people communicate. Gear, pulley, recipe, oil-gas, and other ratios are written in simplest form. Otherwise any number of equivalent ratios could be used and a person might not recognize them as equivalent.

DIVISION METHOD

This method depends upon the idea that division by 1 does not change the value of a number; and that a number divided by itself (except 0) is 1.

For instance: $15 \div 1 = 15$

$\frac{5}{5} = 1$

Example 1:

Simplify $\frac{12}{18}$

$\frac{12}{18} \div 1 = \frac{12}{18} \div \frac{6}{6}$

(Since 12 and 18 are both divisible by 6 and $\frac{6}{6} = 1$)

$= \frac{12 \div 6}{18 \div 6}$

$= \frac{2}{3}$

Therefore $\frac{12}{18} = \frac{2}{3}$

FACTORING METHOD

The factoring method depends upon your ability to factor numbers into primes and your understanding of multiplying fractional numbers.

For instance, if $\frac{4}{5} \cdot \frac{3}{3} = \frac{4 \cdot 3}{5 \cdot 3}$,

then we must agree that

$\frac{4 \cdot 3}{5 \cdot 3} = \frac{4}{5} \cdot \frac{3}{3}$.

Example 1:

Simplify $\frac{12}{18}$.

by factoring: $\frac{12}{18} = \frac{2 \cdot 2 \cdot 3}{2 \cdot 3 \cdot 3}$

$\frac{2 \cdot 2 \cdot 3}{2 \cdot 3 \cdot 3} = \frac{2 \cdot 3 \cdot 2}{2 \cdot 3 \cdot 3}$

$= \frac{2 \cdot 3}{2 \cdot 3} \cdot \frac{2}{3}$

$= 1 \cdot \frac{2}{3}$

$= \frac{2}{3}$

Therefore $\frac{12}{18} = \frac{2}{3}$.

DIV. METHOD (cont'd.)

Example 2:

Simplify $\frac{36}{120}$

$$\frac{36}{120} \div 1 = \frac{36}{120} \div \frac{12}{12}$$

$$= \frac{36 \div 12}{120 \div 12}$$

$$= \frac{3}{10}$$

Therefore $\frac{36}{120} = \frac{3}{10}$

FACT. METHOD (cont'd.)

Example 2:

Simplify $\frac{36}{120}$

factoring: $\frac{36}{120} = \frac{2 \cdot 2 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 3 \cdot 5}$

$$\frac{2 \cdot 2 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 3 \cdot 5} = \frac{2 \cdot 2 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 3 \cdot 2 \cdot 5}$$

$$= \frac{2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 3} \cdot \frac{3}{2 \cdot 5}$$

$$= 1 \cdot \frac{3}{10}$$

$$= \frac{3}{10}$$

Therefore $\frac{36}{120} = \frac{3}{10}$.

EXERCISES I

Simplify each of the following ratios.

1. $\frac{6}{8}$	2. $\frac{6}{9}$	3. $\frac{8}{10}$	4. $\frac{15}{21}$	5. $\frac{8}{12}$
6. $\frac{24}{30}$	7. $\frac{8}{9}$	8. $\frac{14}{28}$	9. $\frac{18}{12}$	10. $\frac{20}{24}$
11. $\frac{12}{16}$	12. $\frac{16}{24}$	13. $\frac{10}{15}$	14. $\frac{8}{15}$	15. $\frac{16}{10}$
16. $\frac{9}{21}$	17. $\frac{21}{28}$	18. $\frac{18}{34}$	19. $\frac{35}{20}$	20. $\frac{24}{16}$

EQUIVALENT RATIOS

Ratios can be shown equivalent without using pictures.

MULTIPLICATION METHOD

If both the numerator and denominator are multiplied by the same number, the result is an equivalent ratio.

Example 1:

$$\frac{1}{3}\cdot\frac{2}{2} = \frac{1\cdot 2}{3\cdot 2} = \frac{2}{6},$$

Therefore $\frac{1}{3} = \frac{2}{6}$.

$\frac{1}{3}\cdot\frac{2}{2}$ is equivalent to $\frac{1}{3}\cdot 1$ (since $\frac{2}{2} = 1$). Multiplying a number by 1 does not change the value so an equivalent ratio is obtained.

FACTORING METHOD

If we agree that $\frac{4}{5}\cdot\frac{3}{3} = \frac{4\cdot 3}{5\cdot 3}$, then this must be true:

$$\frac{4\cdot 3}{5\cdot 3} = \frac{4}{5}\cdot\frac{3}{3}.$$

Example 1:

$$\frac{2}{3}\cdot 1 = \frac{2}{3}\cdot\frac{5}{5}$$

$$= \frac{2\cdot 5}{3\cdot 5}$$

$$= \frac{10}{15}$$

This example can be worked "backwards":

$$\frac{10}{15} = \frac{5\cdot 2}{5\cdot 3} \text{ (by factoring)}$$

$$= \frac{5}{5}\cdot\frac{2}{3} \text{ (using the definition of multiplication)}$$

$$= 1\cdot\frac{2}{3}$$

$$= \frac{2}{3}$$

Therefore $\frac{10}{15} = \frac{2}{3}$.

MULT. METHOD (cont'd.)

Example 2:

$$\frac{1}{3} \cdot \frac{6}{6} = \frac{1 \cdot 6}{3 \cdot 6} = \frac{6}{18}$$

Then $\frac{1}{3} = \frac{6}{18}$

Look at examples 1 and 2. If both $\frac{2}{6}$ and $\frac{6}{18}$ are equivalent to $\frac{1}{3}$, then $\frac{2}{6} = \frac{6}{18}$.

Example 3:

$$\frac{4}{6} \stackrel{?}{=} \frac{12}{18}$$

($\stackrel{?}{=}$ means "Are they equal?")

Since $12 \div 4 = 3$,

multiply $\frac{4}{6} \cdot \frac{3}{3} = \frac{12}{18}$

Therefore $\frac{4}{6} = \frac{12}{18}$.

FACT. METHOD (cont'd.)

Example 2:

$$\frac{8}{10} \stackrel{?}{=} \frac{4}{5}$$

$$\frac{8}{10} = \frac{2 \cdot 2 \cdot 2}{2 \cdot 5}$$

$$= \frac{2}{2} \cdot \frac{2 \cdot 2}{5}$$

$$= 1 \cdot \frac{4}{5}$$

$$= \frac{4}{5}$$

Therefore $\frac{8}{10} = \frac{4}{5}$.

Example 3:

$$\frac{12}{18} \stackrel{?}{=} \frac{9}{12}$$

$\frac{12}{18} = \frac{2 \cdot 2 \cdot 3}{2 \cdot 3 \cdot 3}$ and $\frac{9}{12} = \frac{3 \cdot 3}{2 \cdot 2 \cdot 3}$

$$\frac{2 \cdot 3}{2 \cdot 3} \cdot \frac{2}{3} \stackrel{?}{=} \frac{3}{3} \cdot \frac{3}{2 \cdot 2}$$

$$1 \cdot \frac{2}{3} \stackrel{?}{=} 1 \cdot \frac{3}{4}$$

$$\frac{2}{3} \neq \frac{3}{4}$$

DIVISION METHOD

Dividing a number by 1 does not change its value.

Example 1:

$$\frac{10}{15} \stackrel{?}{=} \frac{2}{3}$$

$$\frac{10}{15} = \frac{10}{15} \div 1$$

$$= \frac{10}{15} \div \frac{5}{5}$$

(since 10 and 15 can both be divided by 5)

$$= \frac{10 \div 5}{15 \div 5} = \frac{2}{3}$$

Therefore $\frac{10}{15} = \frac{2}{3}$.

Example 2:

$$\frac{27}{36} \stackrel{?}{=} \frac{3}{4}$$

(27 and 36 can both be divided by 9)

$$\frac{27}{36} \div \frac{9}{9} = \frac{27 \div 9}{36 \div 9}$$

$$= \frac{3}{4}$$

Therefore $\frac{27}{36} = \frac{3}{4}$.

FACT. METHOD (cont'd.)

Example 4:

$$\frac{48}{72} \stackrel{?}{=} \frac{60}{90}$$

$$\frac{48}{72} = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3}$$

and

$$\frac{60}{90} = \frac{2 \cdot 2 \cdot 3 \cdot 5}{2 \cdot 3 \cdot 3 \cdot 5}$$

$$\frac{2 \cdot 2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 3} \cdot \frac{2}{3} \stackrel{?}{=} \frac{2 \cdot 3 \cdot 5}{2 \cdot 3 \cdot 5} \cdot \frac{2}{3}$$

$$1 \cdot \frac{2}{3} \stackrel{?}{=} 1 \cdot \frac{2}{3}$$

$$\frac{2}{3} = \frac{2}{3}$$

Therefore $\frac{48}{72} = \frac{60}{90}$.

EXERCISES 2

Choose the method you want to use and show which of the following pairs of ratios are equivalent.

Circle the pairs of ratios that are equivalent.

1. $\frac{6}{9} \stackrel{?}{=} \frac{12}{18}$
2. $\frac{6}{8} \stackrel{?}{=} \frac{18}{24}$
3. $\frac{24}{30} \stackrel{?}{=} \frac{12}{15}$
4. $\frac{12}{18} \stackrel{?}{=} \frac{2}{3}$
5. $\frac{2}{5} \stackrel{?}{=} \frac{8}{20}$
6. $\frac{15}{18} \stackrel{?}{=} \frac{5}{6}$
7. $\frac{9}{12} \stackrel{?}{=} \frac{36}{46}$
8. $\frac{24}{30} \stackrel{?}{=} \frac{8}{10}$
9. $\frac{7}{4} \stackrel{?}{=} \frac{36}{46}$
10. $\frac{8}{12} \stackrel{?}{=} \frac{16}{24}$
11. $\frac{30}{36} \stackrel{?}{=} \frac{10}{13}$
12. $\frac{18}{10} \stackrel{?}{=} \frac{9}{6}$
13. $\frac{4}{9} \stackrel{?}{=} \frac{20}{45}$
14. $\frac{25}{10} \stackrel{?}{=} \frac{5}{2}$
15. $\frac{18}{20} \stackrel{?}{=} \frac{6}{7}$
16. $\frac{28}{20} \stackrel{?}{=} \frac{7}{5}$

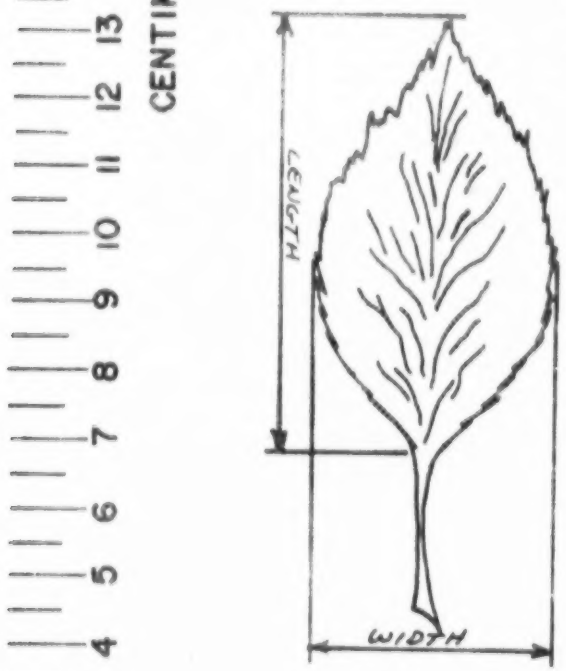
MEASURING LEAVES

CLASS ACTIVITY

For this activity you will be working by yourself.

Read over the activity before beginning to do any measuring.

Equipment Inventory: 1 ruler graduated in half-centimeters (cm.), at least 8 leaves from the same plant.



Activity:

1. Measure the length and width of 4 leaves.

   a. If you do not have real leaves, use those on the next page.

   b. Measure to the nearest half centimeter. Cut the ruler from the left margin of this page.

   c. Record your measurements and complete the table at the bottom of the next page.

AMERICAN ELM LEAVES

Leaf	a	b	c	d
Length (cm.)				
Width (cm.)				
Ratio: $\frac{\text{Length}}{\text{Width}}$				

2. Find the average of the four ratios in this way:

   a. Change each ratio to decimal form by dividing the denominator into the numerator.

   Example:

   $\frac{1}{4}$ can be written in decimal form by dividing

   $$4\overline{)1.25}$$
   $$\begin{array}{r} 5.00 \\ 4 \\ \hline 10 \\ 8 \\ \hline 20 \end{array}$$

Leaf	a	b	c	d
Ratio				
Decimal				

   b. Add the 4 decimal values = _____

   c. Average: Sum ÷ 4 = _____

3. Measure the widths of 4 other leaves from the same plant used in part 1.

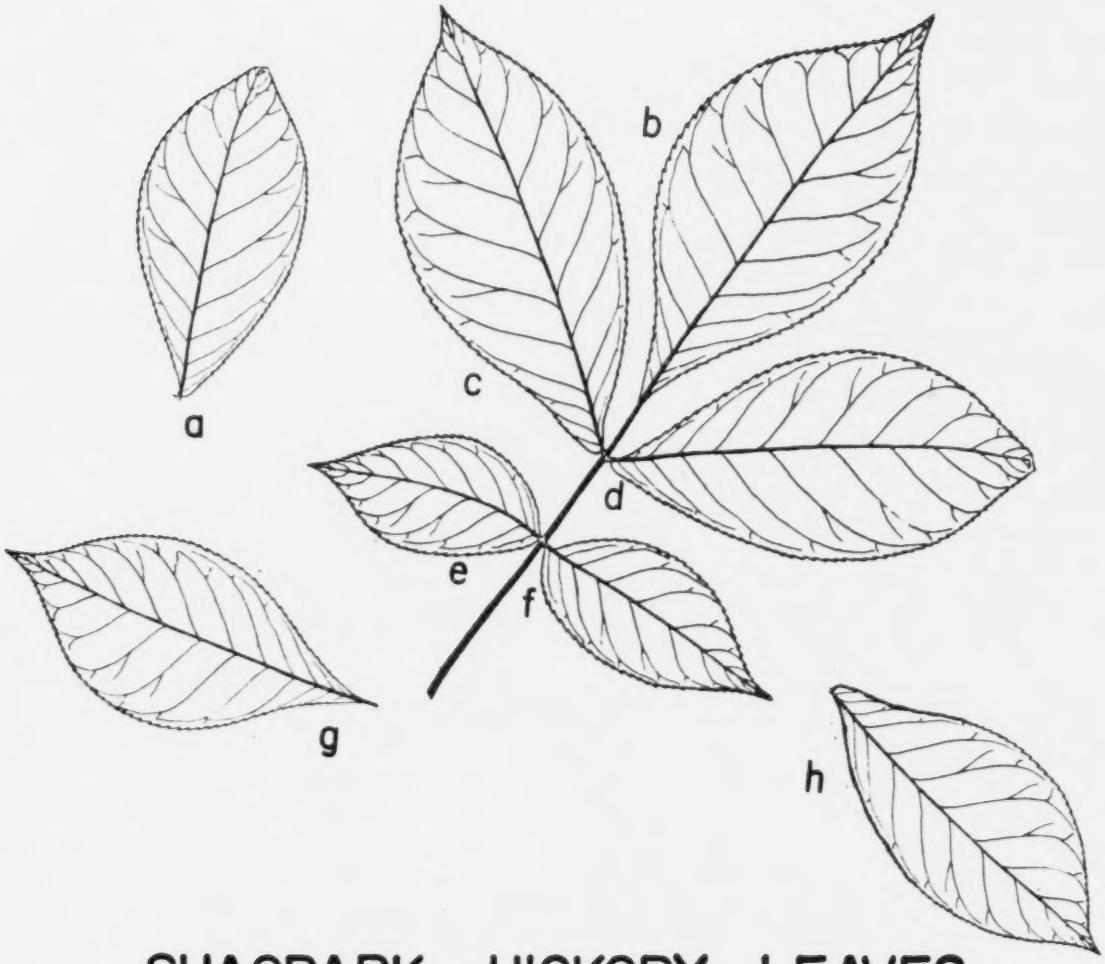
Leaf	e	f	g	h
Width (cm.)				

4. Predict the lengths of the 4 leaves using the "average ratio."

   Length = width × average ratio.

Leaf	e	f	g	h
Predicted Length				
Measured Length				

6. Repeat parts 1-4 using the leaves below.



SHAGBARK HICKORY LEAVES

Leaf	a	b	c	d
Length (cm.)				
Width (cm.)				
Ratio: $\frac{\text{Length}}{\text{Width}}$				
Decimal				

Leaf	e	f	g	h
Measured Width				
Predicted Length				
Measured Length				

ROAST BEEF AND THE WEATHER

PROPORTION

A useful tool in solving problems is equivalent ratios. Equivalent ratios written like this, $\frac{2}{3} = \frac{6}{9}$, have another name. The whole expression, $\frac{2}{3} = \frac{6}{9}$, is called a proportion.

A PROPORTION IS A STATEMENT THAT TWO RATIOS ARE EQUIVALENT.

Other examples of proportions are: $\frac{3}{4} = \frac{6}{8}$, $\frac{5}{10} = \frac{10}{20}$, $\frac{3}{1} = \frac{9}{3}$.

Each of the above are true proportions because each is a pair of equivalent ratios.

A <u>false proportion</u> is one where the ratios are not equivalent. Examples are: $\frac{1}{3} = \frac{1}{2}$, $\frac{2}{3} = \frac{4}{8}$, $\frac{3}{5} = \frac{5}{3}$.

✔ POINT

How can you tell whether or not a proportion is a true statement?

EXERCISES I

Circle each true proportion.

1. $\frac{2}{3} = \frac{4}{6}$
2. $\frac{3}{7} = \frac{9}{21}$
3. $\frac{2}{5} = \frac{4}{8}$
4. $\frac{18}{33} = \frac{6}{11}$
5. $\frac{1}{3} = \frac{5}{14}$
6. $\frac{5}{3} = \frac{25}{15}$
7. $\frac{4}{5} = \frac{16}{20}$
8. $\frac{6}{8} = \frac{9}{12}$

9. $\frac{4}{12} = \frac{16}{44}$ 10. $\frac{8}{6} = \frac{12}{9}$ 11. $\frac{18}{26} = \frac{27}{39}$ 12. $\frac{10}{18} = \frac{18}{10}$

13. $\frac{8}{14} = \frac{12}{21}$ 14. $\frac{14}{8} = \frac{7}{3}$ 15. $\frac{15}{20} = \frac{10}{18}$ 16. $\frac{6}{14} = \frac{9}{20}$

Proportions are useful in solving problems and making predictions in experiments. If three members of a proportion are known, the fourth can be found by using equivalent ratios.

Example 1: Find N in the proportion $\frac{2}{3} = \frac{N}{12}$ so that it makes a true proportion.

Remember, a proportion is a statement that two ratios are equivalent.

$\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$, so N = 8.

Multiplying a number by 1 does not change its value. A new form is obtained called an equivalent ratio.

In this case, $\frac{2}{3} \times 1 = \frac{2}{3} \times \frac{\square}{\square} = \frac{2 \times \square}{3 \times \square} = \frac{N}{12}$.

Since $3 \times \square = 12$, the number we want is the solution of $12 \div 3 = \square$ or 4.

$12 \div 3 = 4$, so $\frac{2}{3} \times 1 = \frac{2}{3} \times \frac{\boxed{4}}{\boxed{4}} = \frac{2 \times \boxed{4}}{3 \times \boxed{4}} = \frac{8}{12}$.

Therefore, N = 8.

Example 2: $\frac{4}{7} = \frac{M}{21}$. Since $21 \div 7 = 3$, multiply $\frac{4}{7}$ by $\frac{3}{3}$.

Then $\frac{4}{7} \times \frac{3}{3} = \frac{12}{21}$, so M = 12.

Example 3: If a weight of 5 ounces stretches a spring 3 centimeters, and some unknown weight W stretches the spring 12 cm., how much is W?

A proportion expressing this problem is $\frac{5}{3} = \frac{W}{12}$.

Here 1 is expressed by $\frac{4}{4}$, and W is 20.

Example 4: A weight of 20 oz. stretches a spring 15 cm. What weight W stretches the same spring 3 cm.?

$\frac{20}{15} = \frac{W}{3}$ is the proportion to be solved.

In this case division by 1 can be used.

$$\frac{20}{15} \div \frac{5}{5} = \frac{4}{3}$$

Since $15 \div 5$ is 3 and dividing by 1 does not change the value of a number, we must divide 20 by 5 and find out that W = 4.

EXERCISES 2

Solve each proportion for the missing member.

1. $\frac{3}{8} = \frac{N}{24}$ 2. $\frac{4}{5} = \frac{M}{15}$ 3. $\frac{24}{36} = \frac{H}{12}$ 4. $\frac{15}{20} = \frac{30}{E}$

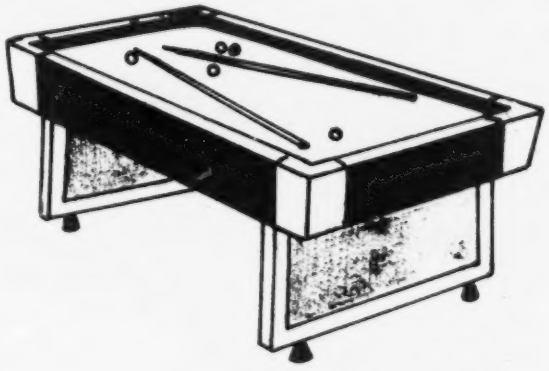
5. $\frac{20}{x} = \frac{5}{3}$ 6. $\frac{R}{10} = \frac{32}{40}$ 7. $\frac{7}{9} = \frac{W}{27}$ 8. $\frac{8}{6} = \frac{32}{T}$

9. $\frac{5}{7} = \frac{N}{35}$ 10. $\frac{12}{20} = \frac{N}{5}$ 11. $\frac{N}{8} = \frac{25}{40}$ 12. $\frac{9}{6} = \frac{N}{36}$

PROPORTIONS AND POOL TABLES

The length and width of a regulation pool table are in the ratio of $\frac{2}{1}$.

A regulation table that is 88" long is 44" wide.



EXERCISES 3

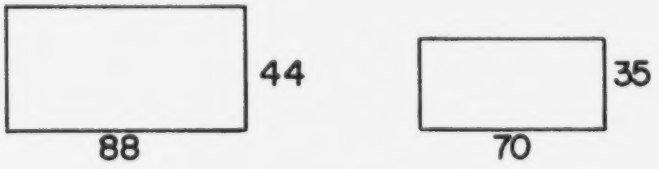
Some measures of other regulation pool tables are given. Find the missing measures.

Table	Length	Width
1	76	
2		41
3	70	

SEVERAL PROPORTIONS

When proportions are used to determine unknown values, there is more than one way to set up the proportion. Not all of them are correct ways.

For example, let's think about pool tables. Since all regulation tables have lengths and widths in the ratio of $\frac{2}{1}$, regulation pool tables are proportional.



A. One way the proportion can be written is

$$\frac{88}{44} = \frac{70}{35}, \qquad \frac{\text{length}}{\text{width}} = \frac{\text{length}}{\text{width}}.$$

B. Another way to write the proportion is

$$\frac{88}{70} = \frac{44}{35}, \qquad \frac{\text{length}}{\text{length}} = \frac{\text{width}}{\text{width}}.$$

EXERCISES 4

Each exercise of the table below contains the measures of two items. They are proportional. One ratio is given and you are to write the second ratio to form a proportion.

Exercise	Item A	Item B	First Ratio	Second Ratio
1	35 miles, 3 gal.	52.5 miles, 4.5 gal.	$\frac{35}{3}$	
2	5 degrees, 30 min.	12 min., 2 degrees	$\frac{5}{2}$	
3	7.5 zips, 4.5 zaps	5 zips, 3 zaps	$\frac{5}{7.5}$	
4	6 ft.long,4 ft. wide	10 ft.wide,15 ft.long	$\frac{4}{6}$	
5	17x, 23y	4.25x, 5.75y	$\frac{5.75}{23}$	
6	30 days, 6 jobs	10 days, 2 jobs	$\frac{2}{10}$	

DISCUSSION QUESTIONS

1. How are the numerator and denominator related for each ratio in proportion A? (Bottom of page 53.)

2. In proportion A, how do the numerators compare? The denominators?

3. How are the numerator and denominator related for each ratio in proportion B? (Bottom of page 53.)

4. In proportion B, how do the numerators compare? The denominators?

5. What is the most important thing you must consider when setting up a proportion?

PROBLEM SOLVING USING PROPORTIONS

Using proportions to solve problems is a valuable tool, and the following exercises will give you practice in setting up proportions.

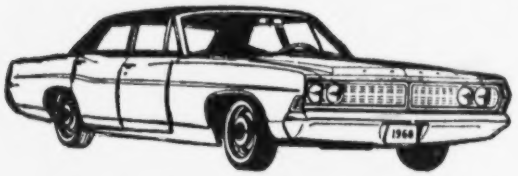
Before writing proportions, you should ask yourself:

(1) What quantities are involved?

(2) What is the unknown quantity?

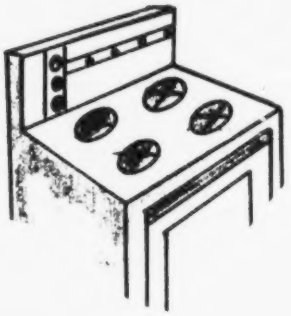
EXERCISES 5

Read each exercise carefully and write only the proportion that you would use to solve the problem. Do not take the time to find the actual answer to the problem.



1. A car travels 96 miles on 6 gallons of gasoline. How far could it go on 24 gal.?

2. Roast beef requires a cooking time of 20 minutes per pound. How many minutes would be required to cook a $5\frac{1}{2}$ pound roast?

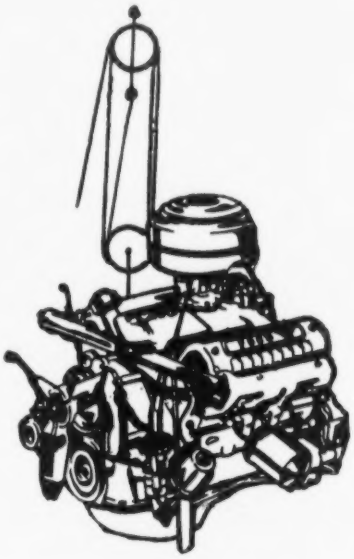


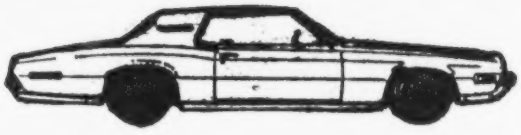
3. One cubic foot of water equals 7.48 gallons and weighs 62.5 pounds. How much does 1 gallon of water weigh?

4. Garden hose varies in size according to the diameter of the opening. A $\frac{3}{4}$-inch hose will deliver 31 gallons per minute. A lawn needs 744 gal. of water for each 1000 sq. ft. of lawn. How many minutes would it take to squirt 744 gal. of water?

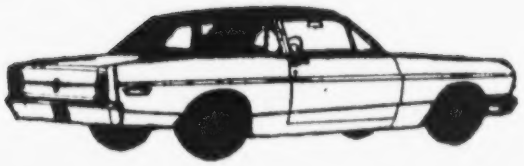
5. A $\frac{1}{2}$-in. hose will deliver only 10.5 gal. per minute. How long would this hose take to produce 744 gallons of water?

6. Pulleys are used to help people lift heavy loads. The pulley system shown works like this: To lift a load of 24 lbs., you would have to exert a 6 lb. pull. How many pounds pull would you have to exert to lift a 600 lb. car engine?

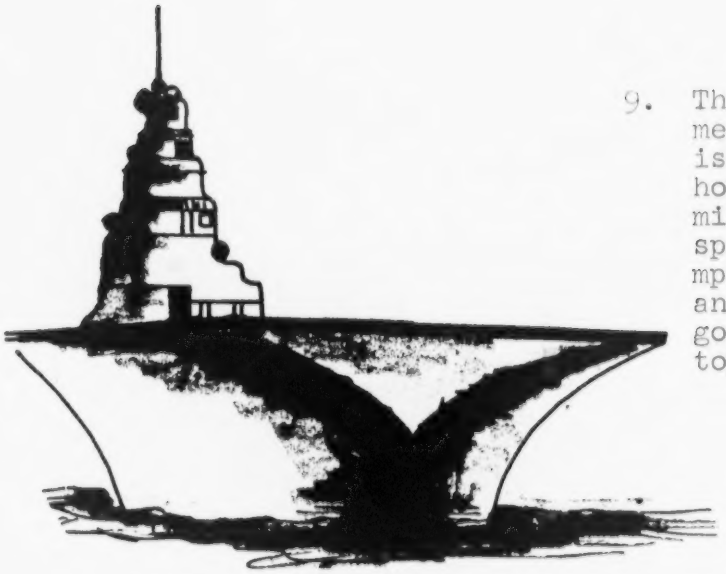




7. A car traveling 60 mph is going at the rate of 88 feet per second. Find the rate of a car in feet per second when it travels 50 mph.



8. Most countries use the metric system for measuring. This means they measure speeds in kilometers per hour. A car traveling 50 mph is going at the rate of 80 km. per hour. What speed in mph would a car be going if it traveled at 120 km. per hour?



9. The speed of ships is measured in knots. A knot is one nautical mile per hour. Nautical and land miles are different. A speed of 20 knots is 23 mph. How many mph will an aircraft carrier be going if it travels at top speed, 30 knots?

10. The marine weather bureau gives wind speed in knots. If they forecast wind speeds up to 55 knots, how many mph would this be?

DISCUSSION QUESTION

Jim Ryan can run the mile in 4 minutes. How long will it take him to run 7 miles? What do you think of the reasonableness of this question?

✓POINT

1. Write the proportion that would be used to solve each problem.

   a. The Imperial gallon is used in Canada. It is larger than the U.S. gallon. For every 5 Imperial gallons, there are 6 U.S. gallons. 17 gallons of gasoline purchased in Canada would be equal to how many gallons in the U.S.?

   b. United States Nickels are made of copper and nickel. For every 3 oz. of copper, 1 oz. of nickel is used. If 14 pounds of copper are to be used for making Nickels, how much nickel is required?

2. Circle the pairs of ratios that are equivalent.

   a. $\frac{4}{9} = \frac{20}{45}$ b. $\frac{15}{10} = \frac{18}{12}$ c. $\frac{14}{18} = \frac{35}{45}$

3. Circle the proportions that are true.

   a. $\frac{9}{6} = \frac{12}{10}$ b. $\frac{12}{18} = \frac{8}{12}$ c. $\frac{7}{11} = \frac{28}{42}$

4. Simplify the following ratios:

   a. $\frac{10}{18}$ b. $\frac{24}{16}$ c. $\frac{28}{36}$

5. Solve these proportions for N.

   a. $\frac{5}{8} = \frac{N}{40}$ b. $\frac{6}{10} = \frac{24}{N}$ c. $\frac{12}{N} = \frac{9}{6}$

THE BOUNCER

Several students will demonstrate this activity for the whole class. If you are one of those who perform this activity, prepare yourself and do a good job. If you are watching the demonstration, think about how an activity should be prepared for, how it should be carried out, and be ready to participate in the summary of the activity.

CLASS ACTIVITY

Read over the activity before beginning the demonstration.

Tell the rest of the students what the activity is about.

The purpose of this activity is to determine the height a ball will bounce when dropped from various distances from the floor. Use this information, in a proportion to predict the height the ball will bounce when dropped from other positions.

Equipment Inventory: 1 ball (golf ball, tennis ball, super ball, etc.)

1 yardstick

Activity:

1. Hold the yardstick against the wall perpendicular to the floor with the 0-inch mark at the floor.

2. Drop the ball from a Height (H) of 40 inches. How far did it rise on the first bounce? _____

3. Drop the ball from 40 inches several times and record your best judgment of the height (h) it bounces. _____

4. Write the ratio, $\frac{H}{h}$ ($\frac{\text{Height ball dropped from}}{\text{height ball bounces}}$): _____

5. Use proportions to predict the missing values in the table.

Height dropped from, H inches	20	50	72
Predicted h after first bounce			

6. Check your results by performing the experiment.

Height, H inches	20	50	72
Experimental results, h			

7. Use proportions to predict the missing values in the table. Note: In this part you are given the height of the bounce, and you are predicting where the ball was dropped from.

height after first bounce	30	45	60
Predicted H			

8. Check your results by performing the experiment.

height h, inches	30	45	60
Experimental results, H			

9. What effect might a different kind of flooring or different ball have on the amount of bounce?

THE CROSS-PRODUCT

The Cross-Product is used to solve proportions when one number is unknown. It is also used to test whether or not a proportion is true.

Some proportions do not have easy numbers to work with, like $\frac{6}{9} \stackrel{?}{=} \frac{14}{21}$. It is not an easy job to find out what number multiplied by 9 equals 21.

So we will look at another way to test proportions, the Cross-Product.

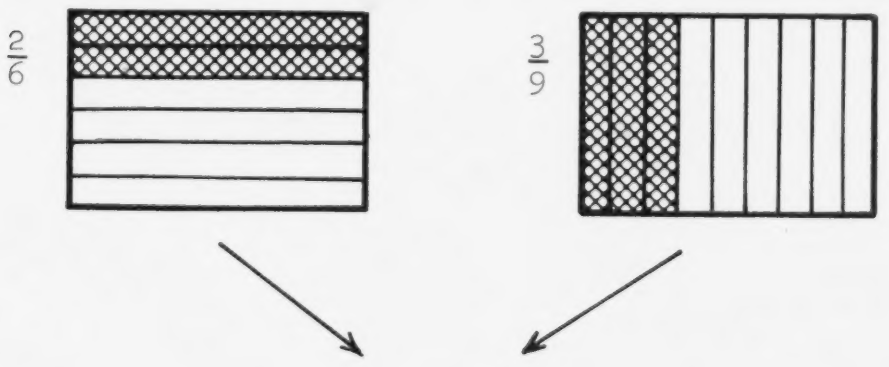
COMPARING RATIOS

Example 1:

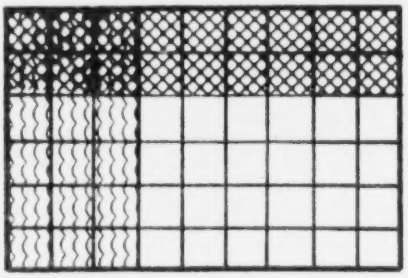
Is the proportion $\frac{2}{6} = \frac{3}{9}$ true?

The description of the model given below along with your teacher's explanation should allow you to see how the model works.

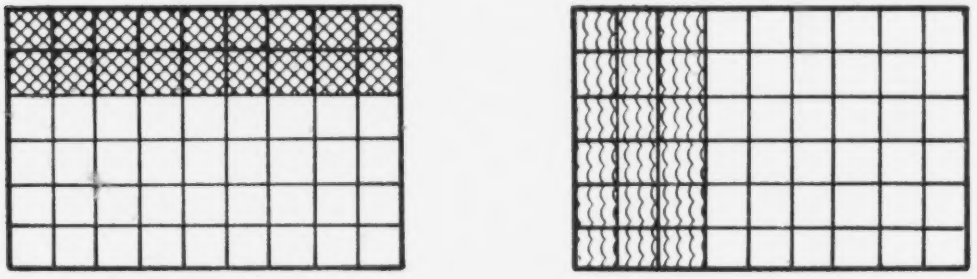
1. Shade two rectangles; each to represent one of the ratios. (The rectangles are the same size.)



2. Slide the two rectangles together so they coincide. The lines from the first rectangle are to be transferred to the second, and the lines from the second to the first.



3. Separate the two rectangles and write the ratio each represents now.



$\frac{18}{54}$ $\frac{18}{54}$

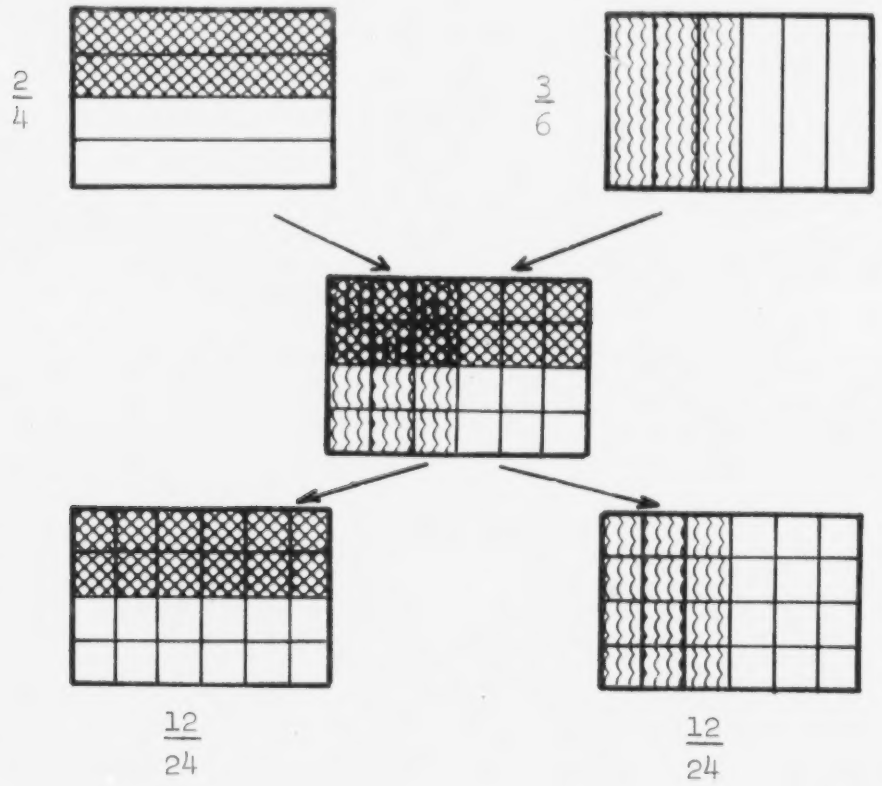
4. The rectangles are now cut into the same number of regions, 54. The shaded areas have different shapes, but each is cut up into 18 small regions of equal size.

Each rectangle represents $\frac{18}{54}$. Since the rectangles are the same as at the beginning except for the number of regions drawn, $\frac{2}{6} = \frac{3}{9}$.

Example 2:

$$\frac{2}{4} \stackrel{?}{=} \frac{3}{6}$$

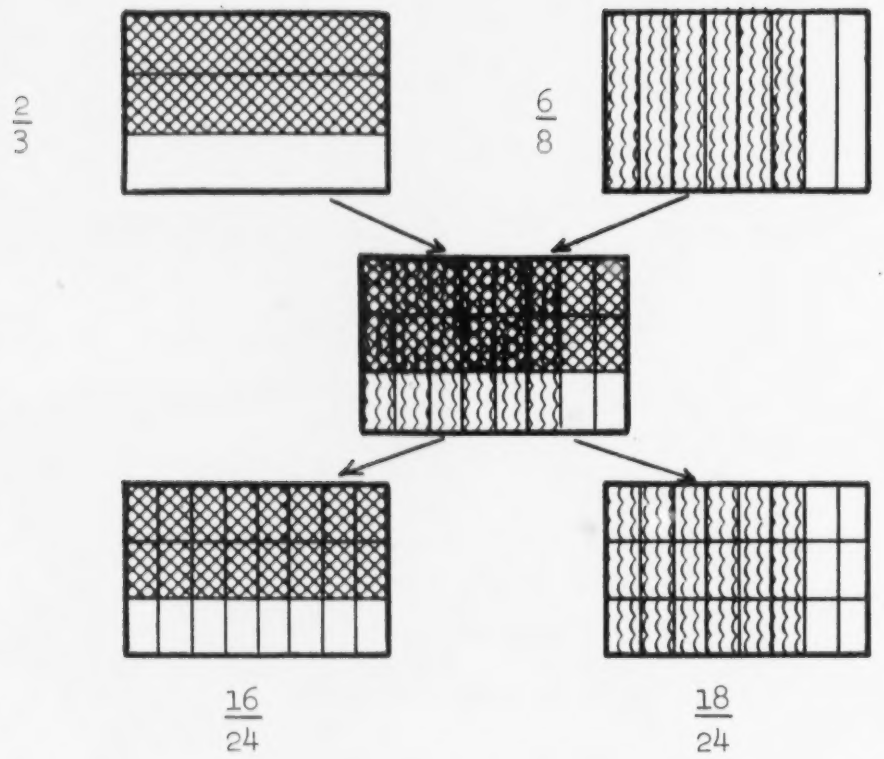
Draw rectangles to represent each of the ratios, slide them together, and pull them apart with each rectangle containing the horizontal and vertical lines.



Then $\frac{2}{4} = \frac{3}{6}$

Example 3:

$\frac{2}{3} \stackrel{?}{=} \frac{6}{8}$



Since $\frac{16}{24} \neq \frac{18}{24}$, the proportion $\frac{2}{3} = \frac{6}{8}$ is false.

EXERCISES I

Draw diagrams like those above to determine whether the proportions are true or false. A worksheet can be found on page 129 for you to show your work.

1. $\frac{2}{3} = \frac{6}{9}$

2. $\frac{2}{3} = \frac{4}{6}$

3. $\frac{4}{6} = \frac{6}{8}$

4. $\frac{3}{4} = \frac{6}{8}$

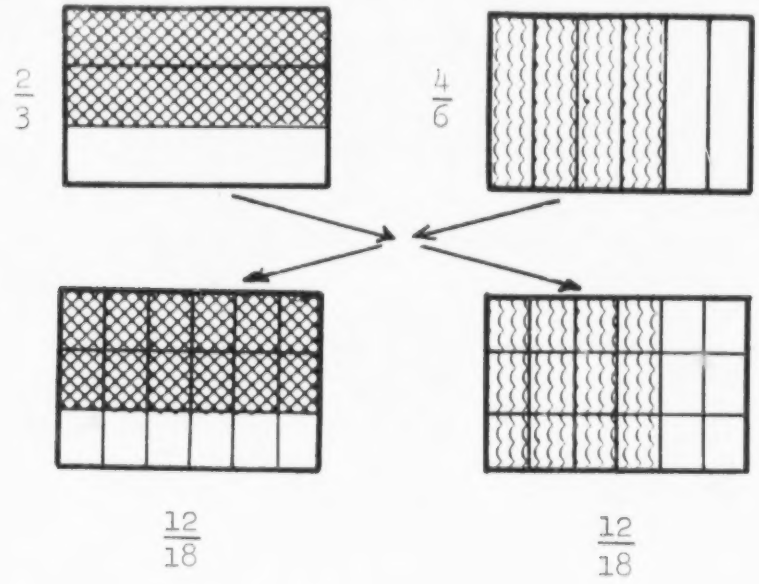
5. $\frac{3}{6} = \frac{4}{8}$

THE CROSS-PRODUCT MODEL

The rectangle model can be used to illustrate the idea of cross-product. The cross-product is a quick way to calculate whether or not a proportion is true.

Example 1:

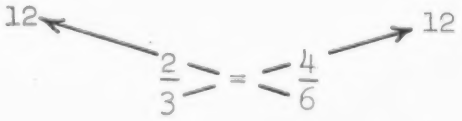
You found out in Exercises 1 that the proportion $\frac{2}{3} = \frac{4}{6}$ is true. The rectangles looked like this.



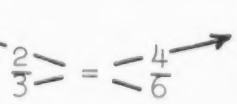
The shaded portion of the left hand rectangle is 2 units by 6 units (2×6). The shaded portion of the right hand rectangle is 3 units by 4 units (3×4).

Notice that $2 \times 6 = 12$ and $3 \times 4 = 12$. Twelve is the numerator of both ratios above.

Also notice



The length and width of the shaded portion of one rectangle.

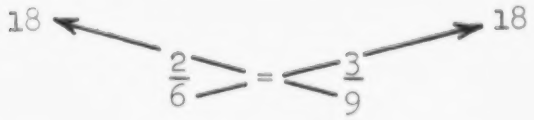


The length and width of the shaded portion of one rectangle.

Multiplying as shown by the arrows is a quick way to find the numerators of the ratios, which are both $\frac{12}{18}$.

Example 2:

Look at example 1 on page 62. $\frac{2}{6} \stackrel{?}{=} \frac{3}{9}$

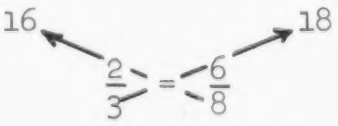


Since both products are 18, the ratios are equivalent and the proportion is true.

18 is the numerator of the ratio $\frac{18}{54}$.

Example 3:

Look at example 3 on page 65. $\frac{2}{3} \stackrel{?}{=} \frac{6}{8}$



Since the products are unlike, 16 and 18, $\frac{2}{3} \neq \frac{6}{8}$.

16 and 18 are the numerators of $\frac{16}{24}$ and $\frac{18}{24}$.

GIVEN A PROPORTION, IF THE CROSS-PRODUCTS ARE EQUAL, THEN IT IS A TRUE PROPORTION. IF THE CROSS-PRODUCTS ARE NOT EQUAL, THEN THE PROPORTION IS FALSE.

The "cross-product" method can be used to test whether or not a proportion is true. (This means that you can test whether or not two ratios are equivalent.)

EXERCISES 2

Find out if the following proportions are true or false. Circle the true proportions.

1. $\frac{4}{7} = \frac{20}{35}$
2. $\frac{8}{10} = \frac{12}{15}$
3. $\frac{4}{6} = \frac{6}{8}$
4. $\frac{10}{4} = \frac{30}{10}$
5. $\frac{9}{12} = \frac{15}{20}$
6. $\frac{8}{5} = \frac{40}{26}$
7. $\frac{10}{15} = \frac{35}{52}$
8. $\frac{14}{20} = \frac{35}{50}$
9. $\frac{24}{27} = \frac{16}{18}$
10. $\frac{36}{21} = \frac{16}{9}$
11. $\frac{8}{3} = \frac{32}{20}$
12. $\frac{15}{27} = \frac{35}{63}$
13. $\frac{12}{21} = \frac{7}{12}$
14. $\frac{14}{9} = \frac{48}{32}$
15. $\frac{6}{15} = \frac{15}{37.5}$
16. $\frac{14}{8} = \frac{35}{22}$

✓ **POINT**

How is the cross-product used to show whether or not a proportion is true?

SOLVING PROPORTIONS USING THE CROSS-PRODUCT

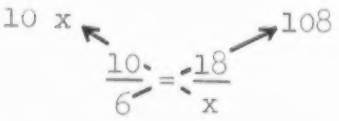
Some proportions that come from experiments and other problems contain numbers quite difficult to work with if you do not use the cross-product method for solution.

Example 1:

If a certain rubber ball bounces 6 inches when dropped from a height of 10 inches, how far will it bounce when dropped from 18 inches?

$\frac{10}{6} = \frac{18}{x}$ There is no convenient number to multiply by 10 to get 18.

Using the cross-product method:



$$10 \cdot x = 6 \cdot 18$$
$$10 \cdot x = 108$$
$$x = \frac{108}{10}$$
$$x = 10.8 \text{ inches}$$

The ball bounces 10.8 inches when dropped from 18 inches.

Example 2:

Solve for N. $\frac{3}{5} = \frac{N}{32}$

$$5 \cdot N = 3 \cdot 32$$

$$5N = 96$$

$$N = \frac{96}{5}$$

$$N = 19\frac{1}{5} \text{ or } 19.2$$

EXERCISES 3

Solve these proportions for N:

1. $\frac{2}{6} = \frac{N}{15}$ 2. $\frac{5}{7} = \frac{N}{20}$ 3. $\frac{8}{3} = \frac{N}{20}$ 4. $\frac{7}{10} = \frac{16}{N}$

5. $\frac{18}{8} = \frac{N}{3}$ 6. $\frac{N}{12} = \frac{10}{18}$ 7. $\frac{5}{6} = \frac{N}{20}$ 8. $\frac{10}{18} = \frac{45}{N}$

9. $\frac{25}{75} = \frac{N}{300}$ 10. $\frac{4}{8} = \frac{N}{10}$ 11. $\frac{N}{35} = \frac{40}{100}$ 12. $\frac{6}{8} = \frac{N}{75}$

13. $\frac{18}{27} = \frac{54}{N}$ 14. $\frac{7}{22} = \frac{24.5}{N}$ 15. $\frac{12}{25} = \frac{N}{20.5}$ 16. $\frac{6.5}{20} = \frac{N}{110}$

EXERCISES 4

Go back to page 55 and solve the problems completely by solving the proportions you wrote there.

HUMAN MEASURES

CLASS ACTIVITY

For this activity you will be working in small groups (3 or 4 students per group).

Read over the activity before beginning to do any measuring and check your equipment against the inventory.

Use proportions when making your predictions. This will let you know how well you can use proportions.

Equipment Inventory: 7 feet of brown wrapping paper, 1 meter stick, you, and masking tape.

Activity: The purpose of this activity is to measure humans and find ratios among the measures. Then the ratios are to be used to predict other human measures.

1. Use masking tape to attach the wrapping paper to the wall and floor. Attach it so you can measure your height, arm span, and foot.

2. Measure to the nearest centimeter your:

   a. height _____
   b. arm span _____
   c. length of foot _____

3. Write the ratios:

   a. comparing length of foot to height. _____

   b. comparing arm span to height. _____

4. Write these ratios in decimal form.

   Example: $\frac{5}{4}$ can be written in decimal form by dividing

$$4\overline{)5.00}\quad 1.25$$
$$\underline{4}$$
$$10$$
$$\underline{8}$$
$$20$$

   a. Length of foot to height. _____

   b. Arm span to height. _____

CLASS DISCUSSION

1. Write on the chalkboard the ratios obtained by the boys in parts 4a and 4b above. How do the ratios compare? (Are they close to the same value?)

2. Write the girls' ratios on the chalkboard. How do they compare?

3. Find the average ratio for the boys' and girls' measurements.

	Ratio	
	Length of foot compared to height	Length of arm span compared to height
Boys		
Girls		

EXERCISES

1. A boy has an arm span of 178 cm. How tall do you think he is?

2. How tall do you think a girl is if her foot measures 23 cm?

3. Measure your teacher's arm span, then predict his or her height. Check your prediction by measuring your teacher's height directly.

✓ POINT

1. Use the cross-product method to test whether or not these proportions are true:

   a. $\frac{9}{12} = \frac{15}{20}$ b. $\frac{7}{18} = \frac{21}{52}$ c. $\frac{12}{18} = \frac{16}{24}$

   d. $\frac{20}{15} = \frac{16}{12}$ e. $\frac{24}{36} = \frac{16}{20}$ f. $\frac{21}{15} = \frac{27}{17}$

2. Solve the following proportions for N:

   a. $\frac{5}{9} = \frac{N}{72}$ b. $\frac{6}{14} = \frac{N}{50}$ c. $\frac{22}{18} = \frac{64}{N}$

   d. $\frac{N}{12} = \frac{8}{32}$ e. $\frac{4}{N} = \frac{26}{46}$ f. $\frac{7}{12} = \frac{N}{84}$

3. Write the proportions that would be used to solve the following:

   a. A 28-inch bicycle wheel has a circumference of about 88 inches. What would be the circumference of a 20-inch bicycle wheel?

   b. In a certain pulley system, a mechanic gives a 24-pound pull to lift a weight of 120 pounds. How much pull would be needed to lift an engine weighing 480 pounds?

   c. For a pair of mother and daughter dresses, the girl's pocket measured $3\frac{1}{2}$ inches wide and $4\frac{1}{2}$ inches long. The width of the mother's pocket was $4\frac{1}{2}$ inches. How long was it?

SPRINGS AND BOLTS

CLASS ACTIVITY

There are some stations around the room where you will perform some experiments involving ratios.

After your teacher divides the class into groups,

1. Go to your station.

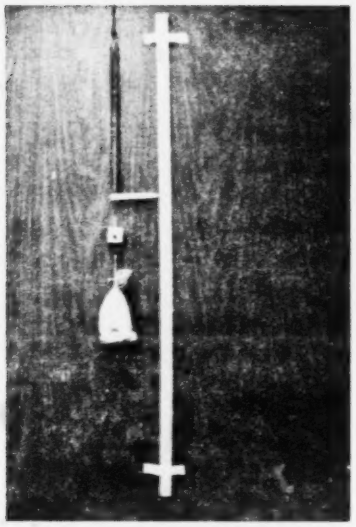
2. Follow the instructions for the activity. They begin on pages 76 and 78.

3. Use the equipment inventory to check your equipment.

4. Record your findings in the spaces provided.

When you finish your first activity, go to a station where the other activity is found.

STRETCHING SPRINGS



Equipment Inventory:

1 spring (S), 1 hook,

1 meter stick, 1 cardboard pointer,

2 known weights (W_1, W_2),

2 unknown weights (W_3, W_4),

1 weight (x).

Activity: This activity centers on the question: How can a spring be used to predict the weight of an object?

1. Using the photograph as a guide,

   a. Place the spring on the hook,

   b. Put the meter stick in a position where measurements can be made,

   c. Attach weight x to the spring to provide some tension.

2. Attach W_1 to the spring or the bottom of weight x.

   How far does the pointer move when weight W_1 is placed on spring, S? _____

3. Remove W_1 and then attach W_2 to the spring.
   What distance does S move when W_2 is used?

4. Write these ratios:
   a. Compare weight W_1 to W_2. _____
   b. Compare the distances moved by W_1 and W_2. _____

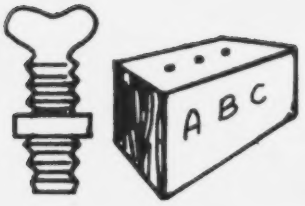
5. How do the ratios in part 4 compare?

6. Attach W_3 to the spring and use a proportion to predict the weight of W_3. _____

7. Use W_4 and S to predict the weight of W_4. _____

8. Use a scale to check the weights of W_3 ____ and W_4 _____.

NUTS AND BOLTS



Equipment Inventory:

1 bolt and nut, 1 block of wood containing three holes, 1 strip of tagboard one inch long.

Activity: This activity centers around the question:

How can a bolt and nut be used to find the depth of a hole in a block of wood?

1. Use the tagboard strip which is one inch long and count the number of threads in one inch of the bolt. _____

2. Turn the bolt in the nut and count the turns as one inch of bolt passes through the nut. _____

3. How far does the bolt move through the nut in one turn? _____

4. If the bolt is turned 6 times, how far does it move? _____

5. Find the depth of holes A, B, C in the wooden block.

a. Start with the end of the bolt even with the nut.

b. Set the nut and bolt over the hole and hold with your fingers.

c. Turn the bolt and count the turns until the bottom of the hole is reached.

Hole	A	B	C
Turns			

d. Calculate how deep the hole is by using the number of turns per inch and writing a proportion.

Hole	A	B	C
depth			

6. A certain screw-type truck jack has 4 threads per inch. How many turns would it take to lift the truck 7 inches off the ground?

7. Look for some information about a micrometer in a mathematics book or an encyclopedia. How are a micrometer and the nut and bolt related in operation?

MAKING COMPARISONS

RATIOS AND COMPARISONS

Proportions are useful when making comparisons of measurements, prices, and performances. One method is to use 100 as a basis for comparison.

Example 1:

During the 1968 baseball season Willie Horton got 146 hits out of 512 at bats while Al Kaline got 94 hits for 327 at bats. A convenient way to compare their performances is to find an equivalent ratio whose denominator is 100 for each player.

$$\frac{146}{512} = \frac{H}{100} \qquad \frac{94}{327} = \frac{K}{100}$$

$$512H = 14600 \qquad 327K = 9400$$

$$H = 28.5 \qquad K = 28.7$$

Their performances were extremely close. Batting at this rate, we could say that Kaline gets 28.7 hits for each 100 times at bat and Horton gets 28.5 hits.

Example 2:

Chef cooking oil comes in two containers like these.

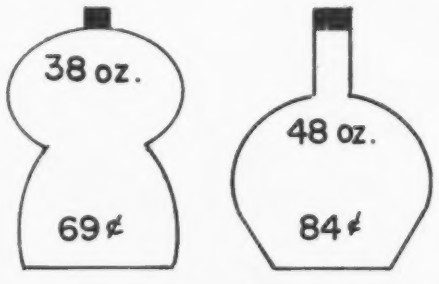
If the oils are of equal quality, which is the better buy?

$$\frac{38}{69} = \frac{X}{100} \qquad \frac{48}{84} = \frac{X}{100}$$

$$69X = 3800 \qquad 84X = 4800$$

$$X = \frac{3800}{69} \qquad X = \frac{4800}{84}$$

$X = 55.1$ oz. for 100¢ $\qquad X = 57.1$ oz. for 100¢



So the 48 oz. jar is the better buy.

EXERCISES I

The following exercises ask you to compare some information. Use proportions to make the comparisons using $\frac{X}{100}$ as the second ratio for each proportion.

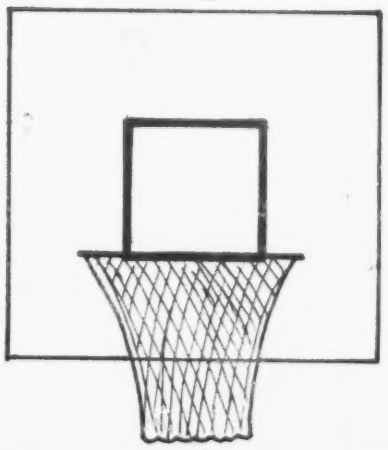
1. An instant coffee called Delight comes in several sizes. As long as spoiling is not a problem, which jar would be the best one to buy?

   A. 14 oz. for $1.79 B. 6 oz. for $.88 (use 179¢ and 88¢)

2. Compare the batting records of the 1968 Detroit Tigers (1292 hits, 5490 at bats) and the 1968 St. Louis Cardinals (1384 hits, 5561 at bats).

3. Among the free throw leaders in professional basketball during 1968 were Cazzie Russell of New York and Jerry West of Los Angeles. Russell made 282 free throws in 349 attempts while West made 391 throws out of 482 tries. Compare their performances.

4. Which would you buy? Delight Coffee: 14 oz. for $1.79, Delight Coffee: 6 oz. for $.88, or Swell Coffee: 10 oz. for $1.19?



5. In January the New York Knicks had won 18 of 47 basketball games. At the same time the Atlanta Hawks had won 17 of 44 games. Use proportions to compare these two records.

RATIOS AND MIXTURES

Nurses and doctors use proportions when preparing medicines. The chemical or drug is not generally used full-strength. Usually a certain amount of drug is mixed with a given amount of liquid to make a solution.

If an alcohol solution is made up of 70 parts alcohol and 30 parts water, the entire solution contains 100 parts. $\frac{70}{100}$ represents the fractional part of the solution that is alcohol.

Example:

Patients may need different amounts of alcohol solutions. These different amounts still have the same strength, $\frac{70}{100}$. A patient needs 30 oz. of alcohol solution. How many ounces of alcohol should the nurse use?

$$\frac{70}{100} = \frac{X}{30}$$

$$100X = 2100$$

X = 21 ounces of alcohol is to be used. Then 9 oz. of water will be added. (30 - 21 = 9)

EXERCISES 2

1. Twelve ounces of a $\frac{25}{100}$ solution of glycerin is to be prepared for a patient. The ratio $\frac{25}{100}$ means that 25 ounces of glycerin is to be used out of every 100 ounces of the solution. (Then 75 oz. of water will be used.) Since only 12 oz. of solution is needed, how much glycerin is required ?

$\frac{25}{100} = \frac{X}{12}$ Finish the problem.

2. How many ounces of boric acid are in 40 oz. of solution if the strength of the solution is $\frac{1}{500}$?

3. If you have 100 oz. of a $\frac{25}{100}$ solution of glycerin,
   a) how many ounces would be glycerin?
   b) how many ounces would be water?

4. If a solution contains 2 parts of a drug and 3 parts of water,

   a) what ratio represents the strength of the solution?
   b) how many ounces of drug would be contained in 100 oz. of solution?

5. Another boric acid solution is stronger, $\frac{55}{100}$.
   a) Out of every 100 oz. of solution, how many oz. of boric acid would be needed?
   b) How many ounces of boric acid would be needed to make 80 oz. of solution?

USING PER CENT

If a nurse knows the strength that is needed for a certain solution, she can use proportions to figure out the number of ounces of drug needed for any given amount of the solution. Using ratios such as $\frac{25}{100}$ or $\frac{55}{100}$ is common when comparing the amount of drug used to the amount of solution being prepared.

The ratio $\frac{55}{100}$ can be read "55 out of 100" or "55 of each hundred". It means that the drug makes up 55 out of 100 parts of the solution. You will recall the special name "per cent" that was used for this kind of comparison.

PER CENT IS A CONVENIENT METHOD OF MAKING COMPARISONS, AS WELL AS GIVING DIRECTIONS FOR MIXING INGREDIENTS (SUCH AS MEDICINES).

A boric acid solution which has a strength of $\frac{55}{100}$ is said to be a 55% solution. The symbol % represents $\frac{}{100}$. Remember: Per cent (%) means "for each hundred". A 55% boric acid solution means 55 oz. of boric acid for each 100 oz. of solution.

If a coat is on sale for 10% off the regular price, it would sell for 10¢ off out of every 100¢ the coat cost ($\frac{10}{100}$). A coat with a regular price of $25.00 (2500¢) would sell for $22.50.

10¢ off — actual number of cents off

$$\frac{10}{100} = \frac{X}{2500}$$

out of each 100¢ — of the 2500¢ regular price.

$100X = 25000$

$X = 250$ cents off regular price

2500¢ - 250¢ = 2250¢ or $22.50 (the sale price).

The symbol, %, is used mainly when writing about per cent and in advertising signs. When you use per cent in solving problems the fraction form is used. In calculating, 25% would be written as $\frac{25}{100}$.

AN IMPORTANT IDEA FOR YOU TO UNDERSTAND IS THAT 30% OR $\frac{30}{100}$ DOES NOT MEAN 100 WAS EXACTLY THE NUMBER OF TIMES AN EVENT TOOK PLACE OR THAN AN ITEM WAS ACTUALLY DIVIDED INTO 100 PARTS..

For instance, if a batter hits at a 30% pace ($\frac{30}{100}$) it does not necessarily mean that he was at bat 100 times. 100 is used because it is a convenient number to compare to and EVERYBODY does it! If a batter gets 6 hits for 20 tries at bat, he also hits 30%.

$$\frac{6}{20} = \frac{X}{100}$$

$$20X = 600$$

$$X = 30 \qquad \frac{30}{100} = 30\%$$

The idea can be illustrated another way. A batter hits safely 5 out of 12 times. What per cent did he hit? This question is asking: "How many hits would he get out of 100 at bats?

$$\frac{5}{12} = \frac{X}{100}$$

$$12X = 500$$

$$X = 41.7$$

41.7 hits out of 100 times at bat is 41.7%. Well! You know that you can't get part of a hit, so he must not be able to be at bat 100 times and still have a batting percentage of 41.7%. Yet, this is still a good way to compare one batter to another. How is a batter's performance reported in the newspaper?

EXERCISES 3

Write the following ratios as per cents using the symbol, %.

1. $\frac{38}{100}$ 2. $\frac{25}{100}$ 3. $\frac{87}{100}$ 4. $\frac{65}{100}$ 5. $\frac{22\frac{1}{2}}{100}$ 6. $\frac{3}{100}$

Write these per cents as ratios.

7. 23% 8. 45% 9. 50% 10. 76% 11. 99% 12. 7%

13. How many ounces of glycerin would be added to 80 oz. of water to get a $\frac{20}{100}$ or 20% glycerin solution?

14. If a medicine is a 25% solution, how many oz. of drug is used to make 24 oz. of solution?

15. A solution is a 25% solution. Does this mean that you have 100 oz. of the solution? Explain.

DISCUSSION QUESTIONS

1. The basketball team made 65% of its free throw attempts.
   a. Write the per cent as a ratio.
   b. Discuss the question: How many free throws did the team attempt?
2. The basketball team had a field goal shooting record of 42.5%. Discuss the number of field goals made by the basketball team.
3. Why is 100 often used as a basis for comparing two situations, such as prices or athletic performances?
4. When 100 is used as a basis for comparisons, sometimes the result is half a person or half a time at bat. If this happens, how can 100 be used for comparison?

STRETCHING RUBBER BANDS

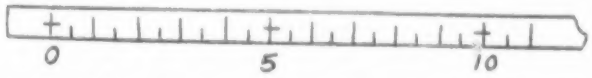
It is helpful to be able to estimate per cents such as the following: 75% of a line segment, 30% of a group of students, 25% of a length of cloth, etc. To help you see per cent, you are going to make a per cent calculator and work with line segments.

CLASS ACTIVITY

A rubber band can be made into a per cent calculator. With this device you can find per cents of line segments and be able to see what they are.

1. To make the calculator you will need a piece of rubber band.

2. Choose a partner. Stretch the rubber band the length of the ruler which is printed along the edge of this page. Then have your partner put a pencil mark on the rubber band every half inch.



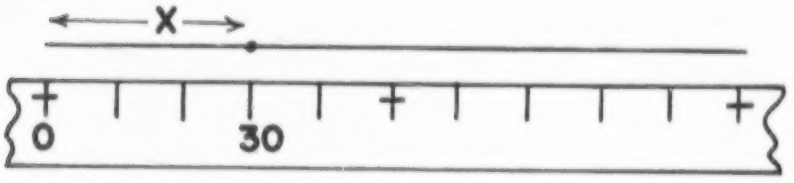
3. When you let the rubber band relax, there is a set of evenly spaced marks. They will stay evenly spaced for any length the rubber band is stretched.

   Let's try it. Mark off 30% of the segment drawn below.

Here's how.

a. Stretch the rubber band next to the segment until it is divided into 10 units of length. (Each unit represents 10%, and there will be some marks on the rubber band that will not be used.)

b. Place a dot on the segment after the third unit which will be the fourth mark on the rubber band.



c. The x on the segment indicates that the left hand part of the segment is 30% of the entire length.

4. Imagine each space having 10 smaller divisions. Then each small space represents 1 out of 100 or 1%. The fourth mark (at the end of the third unit) would then represent 30 out of 100 parts or 30%.

EXERCISES I

1. Place a mark on each segment so that the left hand part represents the given per cent.

a. 50% _____

b. 40% _____

c. 60% _____

d. 25% _____

e. 75% _____

f. 85% _____

g. 90% _____

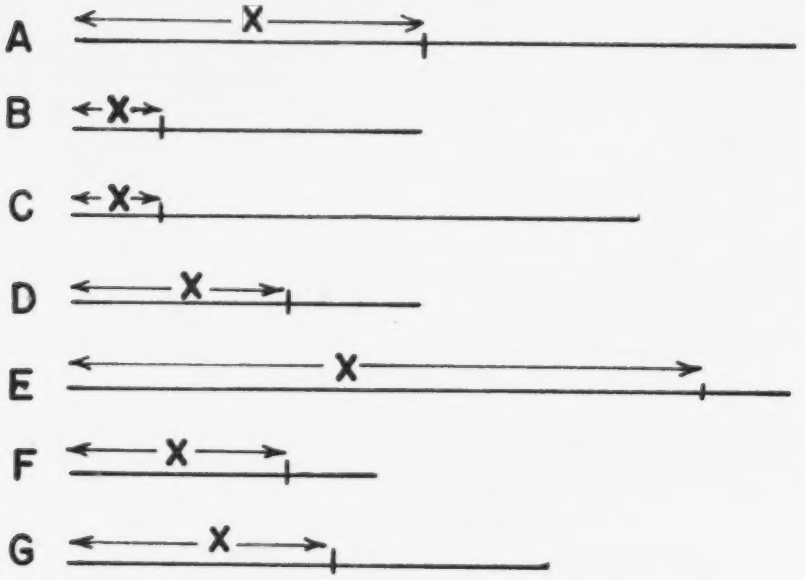
h. 100% _____

i. 125% _____

2. (I) Estimate the per cent represented by the part of the segment indicated by |←— x —→|.

(II) Check your estimate using the rubber band.

	A	B	C	D	E	F	G
I ESTIMATE							
II CHECK							



There are several ways in which a particular per cent can be written. Sometimes one particular form is easier to work with than others.

You know that 25% can be written as $\frac{25}{100}$.

1. If the ratio is simplified you get $\frac{25}{100} = \frac{1}{4}$. So $\frac{1}{4}$ of \$20 is the same as 25% of \$20 which is \$5.

2. To change a fraction into per cent form, use a proportion. For instance $\frac{2}{5} = \frac{x}{100}$, $x = 40$ so $\frac{2}{5} = \frac{40}{100} = 40\%$.

3. Per cent can be represented in decimal form.

$$\frac{25}{100} = 25 \div 100 = .25$$

A simpler way is to read $\frac{25}{100}$ as "twenty-five hundredths" and then write it in decimal form as .25.

4. The forms shown here are equivalent:

$$\frac{40}{100},\ 40\%,\ .40,\ \frac{2}{5}$$

EXERCISES 2

Complete the table by writing the various forms which represent the given per cents.

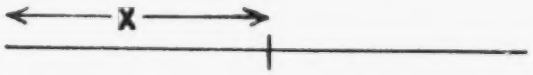
Per Cent form	50%	60%			20%		
Ratio form	$\frac{50}{100}$		$\frac{80}{100}$				
Decimal form	.50					.05	
Simplified fraction form	$\frac{1}{2}$			$\frac{3}{4}$			$\frac{1}{8}$

Some people, when estimating sizes or lengths, think of simplified fractions rather than forms such as decimal or per cent.

When estimating 25% of the length of a segment, a person would most likely think of $\frac{1}{4}$ of the segment (if only for the reason that they have worked more with fractions).

EXERCISES 3

Estimate the per cent or fractional part of the segment by placing a mark that distance from the left end. Indicate the part with an ←x→. Check your estimate using the rubber band.

Example: 50% 

75% _____

25% _____

40% _____

90% _____

3/4 _____

1/3 _____

30% _____

1/4 _____

80% _____

ESTIMATION AND PER CENT

SALES, ESTIMATES, AND SHOES

A sale on shoes! 15% off. Stores often advertise sales in this way because it catches your eye. Some people will buy items even if they do not need them because the idea of "money off" impresses them. Estimation can help a person decide if the amount off is <u>really</u> enough to make purchase worthwhile.

Estimates of per cents can be made more easily if you have some guide for making the estimation. Per cents written in fraction form can be very helpful in estimating because you are quite familiar with fractional parts of numbers. For instance:

$50\% = \frac{50}{100} = \frac{1}{2}$. So 50% of \$25 is $\frac{1}{2} \cdot \$25$ or \$12.50.

EXERCISES I

For each per cent, write a simplified fractional number.

Per Cent	Ratio: $\frac{N}{100}$	Simplified Ratio
50%	$\frac{50}{100}$	$\frac{1}{2}$
75%		
25%		
20%		
40%		
60%		
10%		
30%		

PER CENTS AS SIMPLIFIED RATIOS

As a simplified ratio, 20% = $\frac{1}{5}$ because $\frac{20}{100} = \frac{1}{5}$.

Using 20% as $\frac{1}{5}$, 20% of \$25.00 is \$5.00 because $\frac{1}{5} \times 25 = 5$.

It is not necessary to memorize all of the per cents in a table to use them. Compare 20%, 40%, 60%, and their corresponding ratios $\frac{1}{5}$, $\frac{2}{5}$, and $\frac{3}{5}$. Note that the per cents are multiples of 20 and the ratios are multiples of $\frac{1}{5}$. If you know that 20% = $\frac{1}{5}$, then 60% = $\frac{3}{5}$ because $20 \times 3 = 60$ and $\frac{1}{5} \times 3 = \frac{3}{5}$.

EXERCISES 2

Complete the table by applying the previous discussion.

Per Cent	Simplified Ratio
5%	
10%	
20%	
15%	
25%	
30%	
70%	
40%	
80%	

ESTIMATION...

It is frequently enough to estimate a per cent of a given number. When a sale is advertised, an estimation of the amount of savings is all that is necessary to decide if you want to buy the item or not.

In estimating, the symbol $\approx$ is used and means "approximately the same as".

Example: Estimate 15% of \$25.00.

Solution 1: $5\% = \frac{1}{20}$, so $15\% = \frac{3}{20}$

$\frac{1}{20}$ of 25 is $\frac{1}{20} \times 25 \approx 1$,

so $\frac{3}{20}$ of $25 \approx 3 \times 1 = \$3.00$

Solution 2: 15% is midway between 10% and 20% .

$10\% = \frac{1}{10}$ so $\frac{1}{10} \times 25 = \$2.50$

$20\% = \frac{1}{5}$ so $\frac{1}{5} \times 25 = \$5.00$

This means that 15% of \$25.00 is midway between \$2.50 and \$5.00 which is about \$3.50 . (\$3.75 to be exact.)

Solution 3: Find 10% of \$25.00 and then add half of the amount (5%) to it.
10% of \$25.00 = \$2.50, $\frac{1}{2}$ of \$2.50 = \$1.25.
\$2.50 + \$1.25 = \$3.75.

EXERCISES 3

Estimate the following per cent of the given number.

Given Number	Per Cent	Estimated Amount
18	50%	
24	75%	
35	20%	
20	40%	
70	10%	
90	30%	
120	60%	
120	25%	
80	45%	

Another good use of estimation and per cent is to check whether or not an answer is reasonable.

Example: A $120.00 chair is on sale for 30% off the regular price. What is the amount of savings when buying the chair on sale?

Estimating the amount of savings:

30% is close to 25%

25% = $\frac{1}{4}$

$\frac{1}{4}$ of $120.00 is $30.00

The savings is a little more than $30.00

Calculate the actual amount of savings:

$$\frac{30}{100} = \frac{X}{120}$$

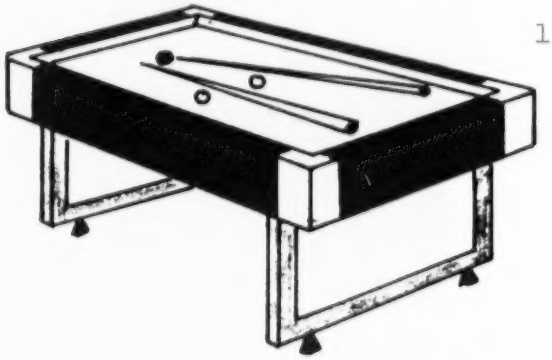
$$100X = 3600$$

$$X = \$36.00 \text{ in savings}$$

If you get an answer of $3.60 or $360.00 your estimate would tell you that these are incorrect.

EXERCISES 4

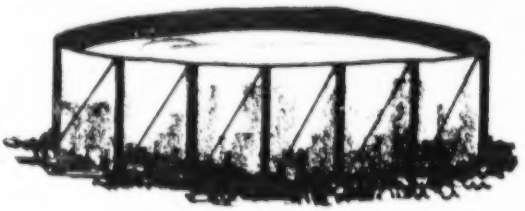
For the following problems, estimate only the answers. Do not actually calculate the correct answer.



1. A pool table regularly selling for $79.95 is advertised on sale for 44% off. What is the amount of savings on this table?

2. How much less does a pool table sell for if it regularly costs $695.00 and is reduced in price by 36% ?

3. These swimming pools are selling for 26% off. How much is saved?
   a) 20 ft. pool, regularly $399.95
   b) 24 ft. pool, regularly $499.95



The speeds of cars and boats cannot be compared directly. Boat speeds are reported in knots while car speeds are in miles per hour.

The way boat owners convert their boat speeds to mph is to use this formula: (Number of knots) + (15% of the number of knots) = mph.

So a boat traveling 20 knots is going 23 mph. (20 knots) + (15% of 20) = 20 + 3 = 23 mph.

15% of 20: 10% of 20 = 2, 5% of 20 = 1,
then 15% of 20 = 3

4. a. Find in mph the speed of a boat traveling 30 knots.

   b. How many mph does a speed of 50 knots represent?

5.

Coats! 20% off. How much would you save?

a) Fur-trim coats: regularly $68.00 and $109.00

b) All-weather coats: regularly $14.99 and $24.99

6.

How much is saved on a one-day sale of ladies' diamond ring sets if they regularly sell for $310.00 and are advertised for 40% off?

7. A rayon instant wardrobe! A coat plus two coordinating shift dresses. They are selling for 30% off the regular price of $23.95. How much would be saved?

8.

The pull of gravity on Mars is 39% of the pull of gravity on Earth.

If you weigh 150 pounds on Earth, what would you weigh on Mars?

9. A ladies' pleated tennis outfit and racket usually sells for $29.99. How much less would it cost when on sale for 20% off?

DISCUSSION QUESTION

When we are estimating costs of items, why doesn't it matter if we do not agree exactly on the estimate?

A CHALLENGE FOR EXPERTS

1.

Most car radiators are protected from freezing by permanent-type antifreeze. The amount of protection is calculated according to the per cent of antifreeze in the cooling system. For instance, if 40% of the coolant is antifreeze, then the car is protected to a temperature of -12° F.

This table shows the protection provided by different per cents of antifreeze.

ADAMS ANTIFREEZE

% of Antifreeze	Your car is Protected to, F
20	+16
25	+10
33	0
40	-12
45	-22
50	-34
55	-48
60	-62

Fill in the table below:

Size of Cooling System, Quarts	Protection Desired, °F	Amount of Antifreeze Required, Quarts
14	-12	
17	-34	
21	0	
16	-48	

2. Adults require 10 mg. (milligrams) of iron per day in their food.

   1000 mg. = 1 gram, 1 gram $\approx$ .04 ounces, so 10 mg. $\approx$ .0004 oz.

   The amount of iron per ounce of cereal varies from brand to brand.

Example: Rice Krispies gives 5% of the adult daily requirement per ounce of cereal. How many mg. of iron is in each ounce of cereal?

Solution using estimation:

10% of the requirement (10 mg.) is found first. This is the same as $\frac{1}{10}$ of that number.

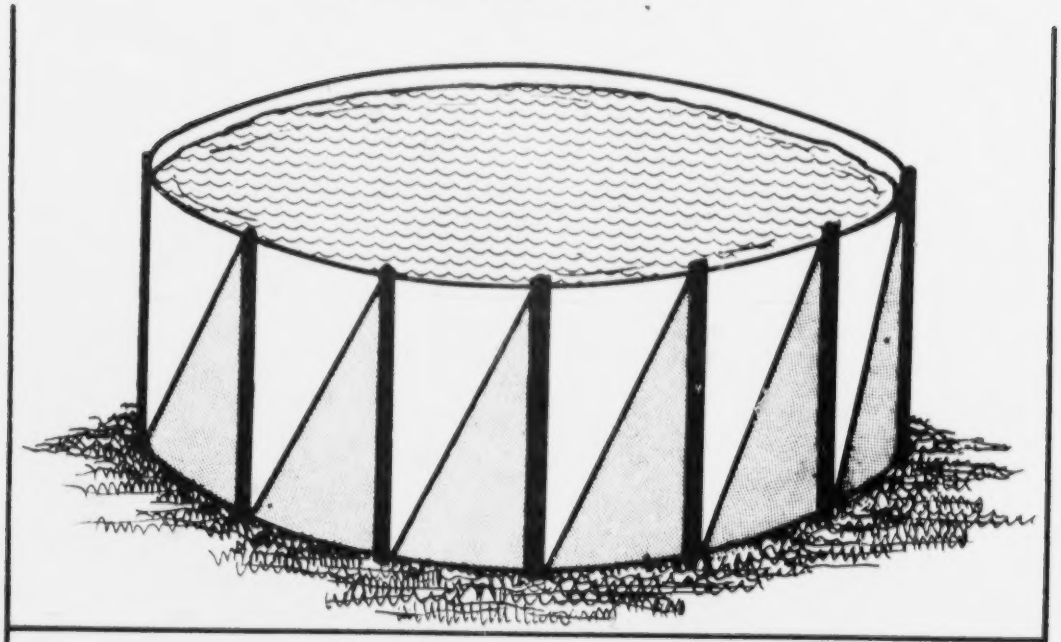
$\frac{1}{10}$ of 10 mg. is 1 mg.

Since 5% is one-half of 10%, Rice Krispies gives an adult .5 mg. of iron per ounce.

Fill in the table:

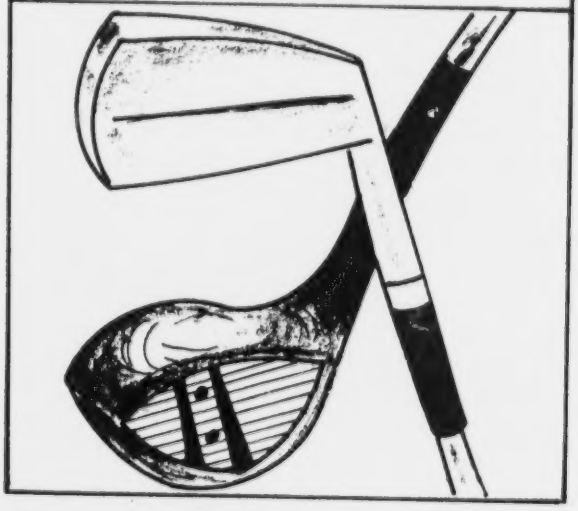
In One Ounce of Cereal	Per Cent of Adult Daily Iron Requirement	Milligrams of Iron Per Ounce of Cereal
Coco-Wheats	8.1%	
Cream of Wheat	120%	
All-Bran	30%	

PUTTERS AND POOLS



SAVE 10% ON POOLS – 5 DAYS ONLY!

SAVE TO 50% OFF GOLF EQUIPMENT



CALCULATING PER CENTS

Proportions can be used to find per cents of numbers.

Example: Find 56% of 283.

Estimation: Finding a per cent of a number is the same as finding a fractional part of the number. 56% is close to 50% or $\frac{1}{2}$.
$\frac{1}{2}$ of 283 is approximately 140.

Solution: $\frac{56}{100} = \frac{N}{283}$

$100N = 15848$

$N = 158.48$ This is close to the estimate.

EXERCISES I

Estimate the answer and then calculate the following percentages.

1. 25% of 88
2. 35% of 120
3. 48% of 130
4. 76% of 90
5. 88% of 64
6. 23% of 44
7. 65% of 150
8. 93% of 165
9. 18% of 58
10. 57% of 234

11. The generally accepted rate of tipping at a restaurant is 15% of the cost of the meal. If you and your date had meals totaling $4.80 at Rick's Restaurant, what tip would you leave for the waitress?

EXERCISES 2

Go back to Exercises 4 on pages 98-101 and find out the actual answers to the questions. Compare your answers to the estimates you made as a check on the reasonableness of your answers.

PER CENT IN ADVERTISING

GOLFER'S PARADISE SALE

MEN'S MATCHED SETS

MODEL	IRON-WOODS	REGULAR PRICE	SALE PRICE
'67 Spalding Silverlines	8-4	$210	$100
'67 Allied Tournaments	8-3	$150	$ 75
'68 Ram Aluminum	9-4	$325	$195
'67 Voit 300 Magnums	9-4	$313	$185
'68 MacGregor Tourney's	9-4	$284	$195
'67 Spalding Elites	8-4	$340	$225
'68 Ram Invitationals	8-3	$180	$ 95
'67 Wilson Signatures	8-4	$270	$150
'68 Wilson Signatures	8-4	$270	$185
'68 Wilson Dave Marr	8-3	$130	$ 70
'68 Spalding Dan Sikes	8-3	$150	$ 88
'68 Burke Bombers	8-3	$200	$115
'67 Slipstreams	9-4	$275	$100
'68 Fernquist-Johnson	9-4	$270	$180
'68 Paul Anthony's	8-3	$100	$ 45
'68 Wilson Shotmakers	8-3	$161	$115

SAVE TO 50% OFF BAGS, CARTS, BALLS, SHOES, etc.

Sale advertisements, especially those involving per cents, should be read carefully.

The amount an item is reduced is called the amount of discount.

For example, the '67 Spalding Silverlines are on sale for $100.00 while they sold regularly for $210.00. The discount is $110.00. When the discount is expressed as a per cent, it is called discount rate or rate of discount.

EXERCISES 3

Read the previous advertisement and answer the questions below.

1. How many sets of golf clubs advertised are on sale at a discount greater than 50%?

2. How many sets are on sale at a discount less than 50%?

3. How many sets are on sale for exactly 50%?

4. On which of the sets advertised will you receive the largest rate of discount? The smallest rate of discount? Be able to justify your answer.

5. On which of the sets advertised will you receive the largest amount of discount? The smallest amount of discount?

6. Compare your answers to exercises 4 and 5. Were your answers the same? Why or why not?

7. What is the meaning of the phrase "save to 50% off"?

8. Do you feel this advertisement is misleading? Why or why not?

SUMMER POOL CLOSEOUTS

ALL POOLS ARE GREATLY REDUCED!!

OPEN 9-9! COME OUT TODAY

DELUXE -- "Our Finest Pool"-- 10-Year Guarantee

SIZE	QUANTITY	SAVE	WAS	NOW
★24'x4'-6"	2	28%	$540	$390
18'x4'-5½"	19	27%	$400	$292
15'x4'-5½"	3	24%	$310	$232

CAPRI --"Lowest Priced Winterized Pool Ever"

SIZE	QUANTITY	SAVE	WAS	NOW
★24'x4'-5½"	41	34%	$420	$277
18'x4'-5½"	3	26%	$310	$229
15'x4'-5½"	13	27%	$250	$182

DELUXE OVAL -- 10-Year Guarantee

SIZE	QUANTITY	SAVE	WAS	NOW
32'x18'x4'	6	29%	$940	$671
25'x15'x4'	2	28%	$730	$522

ARISTOCRAT OVAL -- Includes 2 Redwood Decks

SIZE	QUANTITY	SAVE	WAS	NOW
★31'x16'x4'	3	25%	$700	$526
★24'x12'x4'	11	25%	$540	$405
18'x12'x4'	9	25%	$350	$262
15'x10'x3½'	7	27%	$140	$102

Save an "extra" 10% discount off these prices on any display model pool!

EXERCISES 4

The following exercises refer to the advertisements on the previous page.

1. For the swimming pools in the chart below (they are starred in the ad) calculate the sale price of each pool.

MODEL	SIZE	RATE OF DISCOUNT	REGULAR PRICE	SALE PRICE	ACTUAL CASH SAVINGS
Capri	24' X 4'5½"				
Deluxe	24' X 4'6"				
Aristocrat Oval	31' X 16' X 4'				
Aristocrat Oval	24' X 12' X 4'				

2. Compare the sale prices you calculated to those in the advertisement. Are the prices in the ad exactly what the discount rate indicated?

3. Since the sale price you calculated and the sale they quoted are not equal, the rates of discount in the advertisement are not exact. Why does the ad use discount rates that are not exact?

4. What would be the sale price of a Capri Model 24' × 4' $5\frac{1}{2}$" if the pool were a display model?

✓POINT

1. Using 100 as a basis, compare the following costs for coffee: 2 oz. for 29¢ and 6 oz. for 88¢.

2. Compare the 1968 passing records of two leading football quarterbacks:

   Len Dawson 224 attempts and 131 completions.
   Earl Morrall 317 attempts and 182 completions.

3. A patient needs 40 oz. of medication. The strength of the medicine is 30%. How many ounces of the drug are needed?

4. Write:

   a. $\frac{35}{100}$ as a per cent using the symbol, %.

   b. $\frac{40}{100}$ as a simplified ratio; in decimal form.

   c. .75 as a per cent using the symbol, %.

5. Use a mark to show your estimate of 40% of the line.

---

6. Estimate: a. 15% of 60
   b. 40% of 250

7. Calculate: a. 30% of 145
   b. 45% of 72

8. A store advertised that their $14.95 skirts were on sale for 12% off. How much would be saved on the skirt?

VARIATION

Most of the work you have done with ratio and proportion has involved direct proportion.

DIRECT PROPORTION

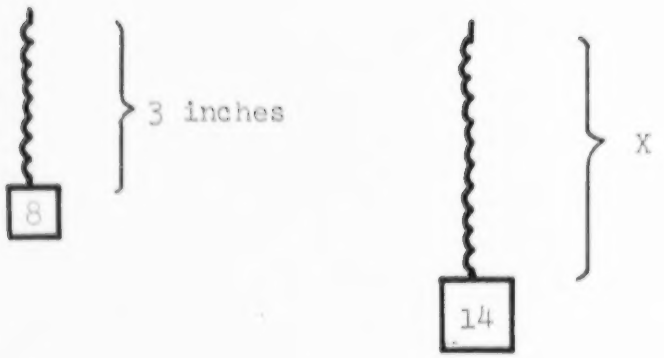
Direct proportion behaves like this: As one quantity increases, a related quantity increases.

Example 1: The price of apples is 2 for 5¢. The more apples you buy, the higher the total price.

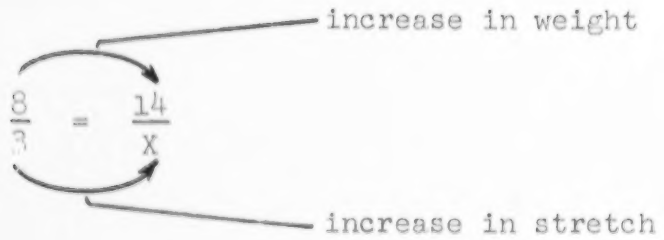
2 apples cost 5¢, 6 apples cost 15¢,
20 apples cost 50¢, etc.

Example 2: The activity, Stretching Springs, involves direct proportion. The more weight you hang on a spring, the longer the spring stretches.

For a particular spring, a weight of 8 ounces stretches the spring 3 inches. How far would a weight of 14 ounces stretch the spring?



One proportion is $\frac{8}{3} = \frac{14}{X}$. From the problem situation you can see that the spring stretches as more weight is added.



EXERCISES I

1. 25 pounds of a certain fertilizer is required for 5000 sq. ft. of lawn. How many pounds of fertilizer would be needed for 9500 sq. ft. of lawn?

2. A set of pulleys requires 5 pounds of pull to lift a weight of 35 pounds. How much pull is required to lift 100 pounds?

INVERSE PROPORTION

Some situations work in a manner opposite to that of direct proportion, and they are called inverse proportions.

An inverse proportion works like this: as one quantity increases a related quantity decreases, or as the first quantity decreases the second quantity increases.

Example: As the age of a car increases, the value of the car decreases.

The key to solving proportion problems is to recognize whether the situation is direct or inverse.

The problem in solving proportion problems is that:

1. Direct proportion problems can be set up in two ways.
2. Inverse proportion problems can be set up in only one way.

Example: Direct Proportion

An alloy of copper and zinc makes brass. The ratio of the weight of copper to weight of zinc is 17 to 13. If 85 pounds of copper is used, how many pounds of zinc will be needed?

One proportion is $\frac{17}{13} = \frac{85}{N}$. $\frac{\text{copper}}{\text{zinc}} = \frac{\text{copper}}{\text{zinc}}$

Another proportion is

$$\frac{17}{85} = \frac{13}{N} \qquad \frac{\text{copper}}{\text{copper}} = \frac{\text{zinc}}{\text{zinc}}$$

$N = 65$ in both cases.

Direct proportions allow two methods to set up the proportion, each resulting in the correct answer.

Examples: Inverse Proportion

1. If 24 men can do a job in 30 days, how many days will it take 36 men to do the job?

   Since more men are working on the job, it will take fewer than 30 days to complete.

   The problem involves an inverse proportion since the number of days decreases as the number of men increases.

An inverse proportion can be set up in only one way. Each ratio must be a comparison of like quantities. In this case men to men and days to days.

The proportion is $\frac{24}{36} = \frac{N}{30}$.

N represents the number of days the 36 men will take to do the job.

Notice that the 24 and 30 (and the 36 and N) are not both in the numerator (in the denominator) although they correspond. This is what makes the proportion inverse. The second ratio appears upside down to what you might expect.

The reason the second ratio is written upside down is explained below.

In $\frac{24}{36} = \frac{N}{30}$, the numerators are smaller than the denominator ($24 < 36$ so $N < 30$).

We know that the number of days the 36 men take will be less than 30, so N must be in the numerator to obtain this result.

One way to set up the proportion is to write the terms so they correspond $\frac{24}{36}$ and $\frac{30}{N}$, then turn the second ratio upside down and write the proportion: $\frac{24}{36} = \frac{N}{30}$.

The number of days needed by the 36 men is $N = 20$.

If a proportion is made where the terms of each ratio are not like quantities, an incorrect answer will result. (You might try it and see.)

2. At 50 mph a car can travel from Huntsville to Carson City in 5 hours. How long will it take to make the trip at 65 mph?

   This is an inverse proportion because as the speed increases the time will decrease.

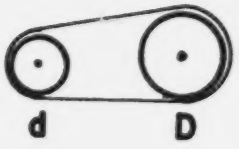
   The proportion is $\frac{50}{65} = \frac{N}{5}$.

   $$65N = 250$$

   $$N \approx 3.8 \text{ hours}$$

EXERCISES 2

1. If the tension of a banjo string is kept constant, the frequency (vibrations per second) is inversely proportional to the length of the string. If a string 30 inches long vibrates 250 vps, how fast will it vibrate when 25 inches long?

2. 

   A small drive wheel (d) is connected by a belt to a larger wheel and drives it (D is the driven wheel).

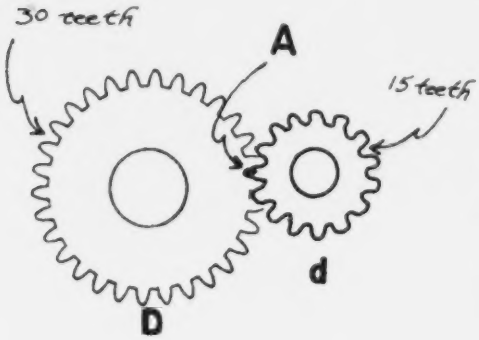
The drive wheel, d, turns faster than the driven wheel, D. The diameter of d is less than the diameter of D.

When two wheels (called pulleys) are connected in this way the number of revolutions per minute (rpm) of a wheel is inversely proportional to the diameter of the wheel.

What is the speed of pulley d if its diameter is 4 inches? D has a diameter of 10 inches and is turning 100 rpm.

3. Find the diameter of a pulley, D, having a speed of 80 rpm if it is driven by a 5 inch pulley having a speed of 200 rpm.

4. Gears behave somewhat like pulleys. Instead of being connected by a belt, they mesh using teeth.



The smaller gear turns faster than the larger gear in order for all of the teeth to mesh, yet the number of teeth in the smaller gear is less than in the larger gear.

a. The teeth meet at point A. If all 30 teeth of D pass point A, how many teeth from d must pass point A?

b. For every revolution of D, how many revolutions does d make?

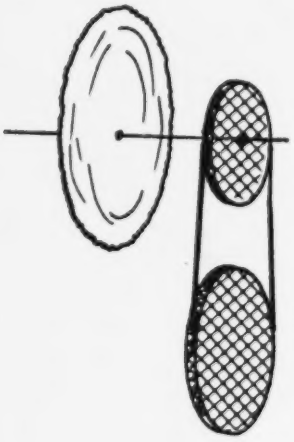
The number of revolutions per minute of two meshed gears is inversely proportional to the number of teeth in the gears.

c. If D turns 100 rpm, how many rpm will d make?

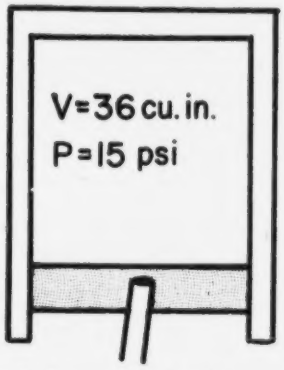
5.

A grinding wheel turns at 850 rpm. It is attached to a 32 tooth gear and is driven by an 8 tooth gear. How fast is the drive gear turning?

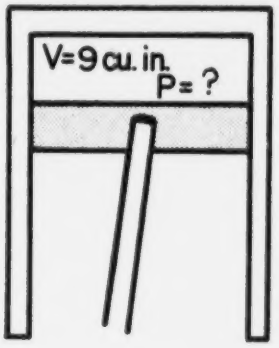
6. A circular saw is connected to a 5 inch pulley and turns 1000 rpm. How large is the drive pulley if it turns 240 rpm?



7. The pressure, P, inside an automobile cylinder is inversely proportional to the volume, V.



piston at bottom of stroke



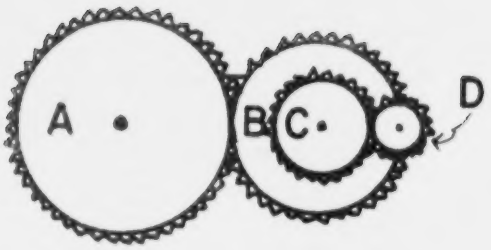
piston at top of stroke

What is the pressure in the compressed cylinder?
(psi means pounds per square inch.)

8. The 1968 Mustang Mach I has a compression ratio of 10.50 to 1. This means that if the volume of the cylinder is 1 when the piston is up, the volume is 10.50 when the piston is at the bottom of the stroke. What is the pressure when the piston is up, if the pressure is 15 psi when the piston is at the bottom of the stroke?

A CHALLENGE FOR EXPERTS

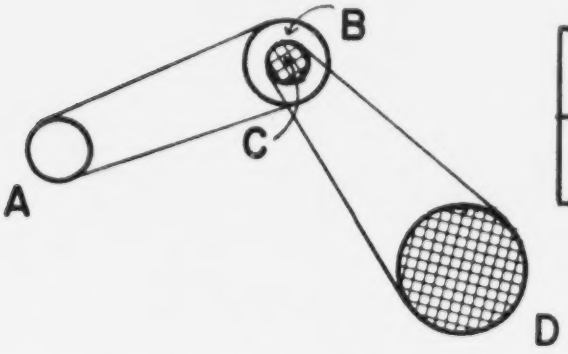
1. The diagram at the right shows a gear train (a train is a series of connected gears.) Gear A is turning at 70 rpm. How many rpm is Gear D turning?



Gear	A	B	C	D
Teeth	90	40	32	20

2. What is the effective gear ratio, $\frac{\text{gear A}}{\text{gear D}}$, in the gear train of exercise 1?

3. What diameter must pulley A have in order for it to turn 2400 rpm? Pulley D is turning 180 rpm.



Pulley	A	B	C	D
Diameter	?	5"	3"	16"

BEATERS AND BEAMS

CLASS ACTIVITY

There are some stations around the room where you will perform some experiments involving ratios.

After your teacher divides the class into groups,

1. Go to your station.
2. Follow the instructions for the activities which begin on the next page.
3. Use the equipment inventory to check your equipment.
4. Record your findings in the spaces provided.

When you finish your first activity, go to a station where the other activity is found.

THE EGG BEATER

Equipment Inventory: 1 Hand egg beater or
1 Hand speed drill

Activity:

This activity centers on the question: How many times does the beater revolve for each complete turn of the handle?

1. Turn the handle one complete turn and have your partner count the number of turns of the beater. _____

2. Turn the handle 4 times. How many beater revolutions were there? ____ For 6 handle turns, how many beater revolutions are there? _____

3. Using the information from parts 1 and 2, how many beater revolutions are there for one handle turn? _____

4. Predict the number of beater turns if the handle makes 8 turns. _____ Did you have to count the turns of the beater? Do you think there is a better way to make this prediction?

5. Write a ratio B to H, where B is the number of beater turns and H is the number of handle turns.

6. Count the teeth in each gear, and write a ratio comparing them. _____

7. How do the ratios you wrote in parts 5 and 6 compare?

8. What is the relation between the handle and beater revolutions and the number of gear teeth on each?

9. What is the precise number of turns the beater will make when the handle is turned one revolution? _____

10. If the beater turns 45 times, how many times will the handle turn? _____

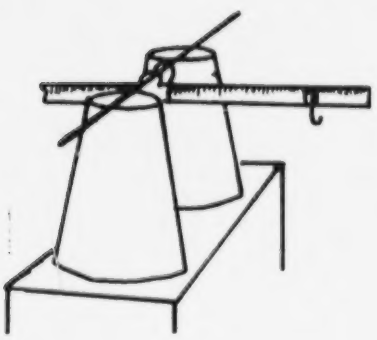
11. Write a proportion which will show how handle and beater turns and number of gear teeth are inversely proportional.

12. An egg beater has gears with 64 and 14 teeth each. If the handle is turned 28 times, how many turns will the beater make?

THE BALANCE BEAM

Equipment Inventory: 1 meter stick, 2 dixie cups, 1 metal paper-clamp, 1 wire rod, 3 known weights (W_1, W_2, W_3), 2 unknown weights (W_4, W_5), 2 metal hooks.

Activity: This activity centers on the question: How can a beam balance be used to predict the weight of an object?



1. Assemble the beam balance including the hooks and move the clamp position until the meter stick balances.

2. Place weights W_1 and W_2 on hooks and slide the hooks until the meter stick balances.

3. Write as ratios:

   a. The distance W_2 is from the center of the stick to the distance W_1 is from the center. $\frac{D_2}{D_1}$:

   b. The weight of W_1 compared to the weight of W_2. $\frac{W_1}{W_2}$:

4. Use weights W_1 and W_3 and repeat parts 2 and 3.

$\frac{D_3}{D_1}$: $\frac{W_1}{W_3}$:

5. Are the ratios obtained in exercises 3 and 4 equivalent?

6. If a proportion is written $\frac{D_2}{D_1} = \frac{W_1}{W_2}$, what kind of a proportion is it? _____ _____

7. Place weights W_1 and W_4 on the beam and adjust until it balances. Use a proportion to predict the weight of W_4. _____

8. Check the weight of W_4 by weighing on a scale. _____

9. Predict the weight of W_5. _____

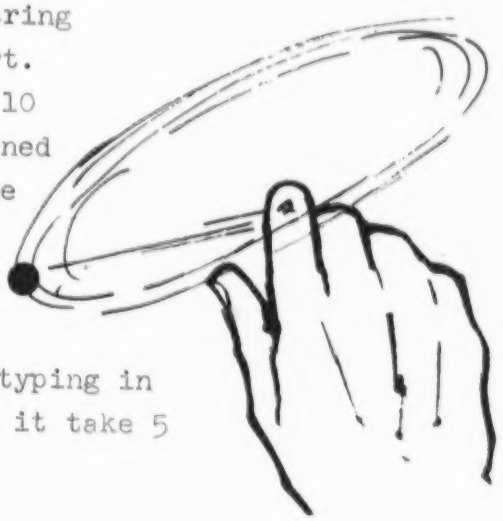
10. Check the weight of W_5. _____

✓ POINT

1. Which of the following situations involve inverse proportions.

   a. Hanging various weights on a spring stretches the spring various lengths. This device can be used to predict the unknown weight of an object.

   b. A ball tied to the end of a string is twirled with constant effort. First the length of string is 10 inches, and then it is lengthened to 15 inches. The speed of the ball changes as the length of string changes.



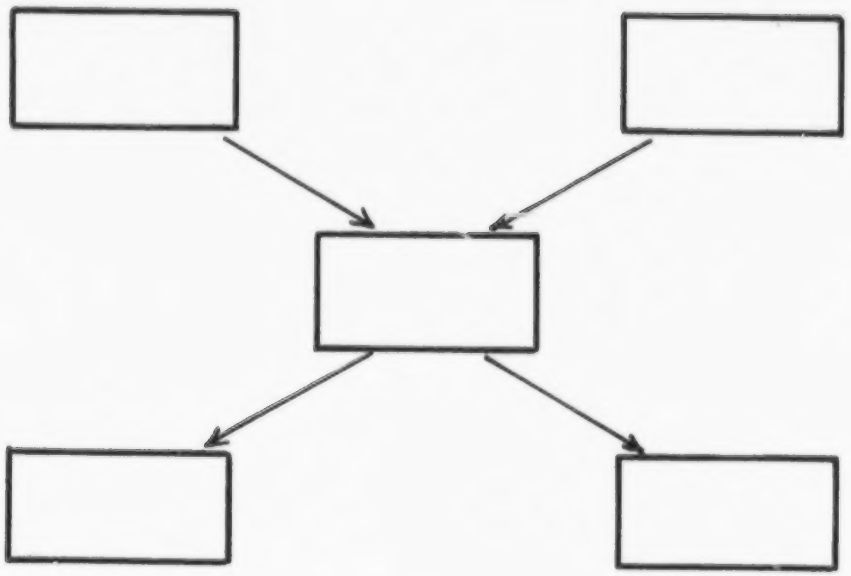
   c. Three secretaries can do some typing in 16 hours. How many hours will it take 5 secretaries to do the job?

   d. A machine can grind 35 bolts in 15 minutes. How many bolts can it grind in 100 minutes?

2. How can you tell whether a situation involves direct or inverse proportions?

Diagram outlines for Exercises 1, page 65.

1.



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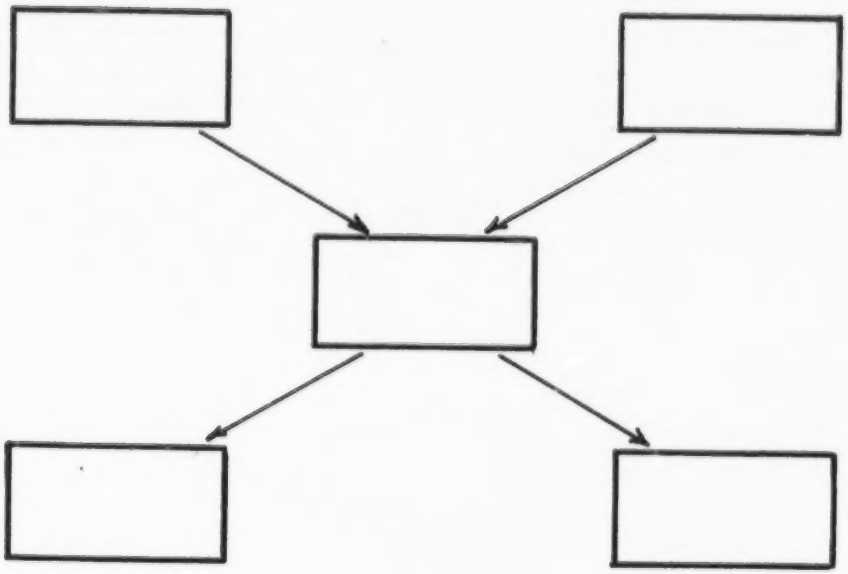

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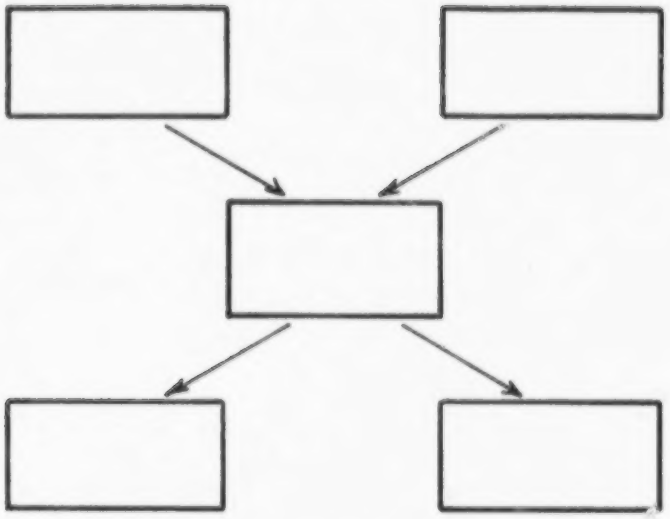

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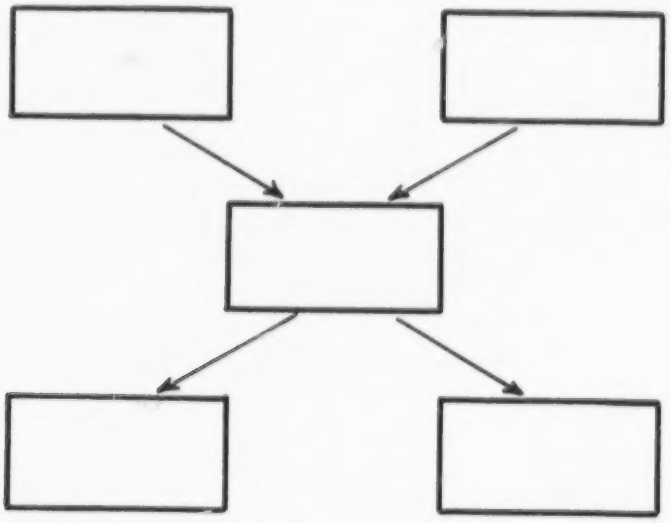

5.